NATURAL INFLATION

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Abstract. It is shown how the inflationary Universe scenario can be implemented in a special class of supersymmetric models without any unnatural fine-tuning of parameters. The special class of models utilizes O'Raifeartaigh-type supersymmetric breaking and employs the Witten "reverse hierarchy" scheme. A prime test example, the Dimopoulos-Raby "geometric hierarchy" model, has been studied in collaboration with A. Albrecht, W. Fischler, S. Dimopoulos, E. Kolb & S. Raby and some results of that study are presented. Implications of this new kind of inflationary model for our understanding of the fluctuations necessary for galaxy formation, the initial cosmological singularity and the very large scale structure of the Universe are also briefly discussed.

Recently, it has been shown by Linde (1982a), Albrecht and Steinhardt (1982a,b) that the Guth (1981) inflationary scenario can be successfully implemented in grand unified theories (GUTs) in which the GUT symmetry is broken through radiative corrections to the tree effective potential — so-called "Coleman-Weinberg (C-W) models" (Coleman and Weinberg, 1973). Unlike the case for ordinary GUTS, (Guth and Weinberg, 1982), the C-W models result in enormous exponential expansion of the Universe during the GUT phase transition without preventing the phase transition from being completed and from homogenizing the Universe. The key conceptual difference is the following: For ordinary GUT inflationary scenarios, the exponential expansion of the Universe occurs while the Universe is supercooling in a metastable (SU(5)) phase; the metastable regions expand so rapidly that bubble nucleation (tunneling) cannot occur rapidly enough for the phase transition to the stable phase to be completed. In C-W GUTs, the major inflation occurs after fluctuations have driven different regions of the Universe a small distance away from the SU(5) symmetric state towards different spontaneous symmetry breaking (SSB) minima; the interiors of the fluctuation regions have a small Higgs expectation value which evolves inevitably towards a stable minimum with large expectation value (due to the potential), but because of the special nature of the C-W potential the evolution occurs very slowly (sometimes referred to as the "slow roll-over" of the scalar field) (Steinhardt, 1982). During the slow roll-over the energy density in the fluctuation region is nearly that of the symmetric phase (because the Higgs expectation value is small) and each fluctuation region begins to expand to a size many orders of magnitude times the size of the observable Universe today. Because the expansion occurs in the evolving stable phase, rather than the metastable phase, the problem of completion of the phase transition that occurred in Guth's original inflationary scenario is obviated. Therefore, the C-W models provide a possible explanation of the cosmological homogeneity, isotropy, flatness and monopole problems (Linde, 1982a; Albrecht & Steinhardt, 1982a, Guth, 1981) and, it has been more recently shown, a natural solution to the problem of baryon asymmetry (Albrecht, et al., 1982a; Abbott, et al., 1982; Dolgov and Linde, 1982) and the problem of providing the fluctuations neccessary for galaxy formation (Chibisov and Mukhanov, 1981; Hawking, 1982; Bardeen, et al., 1982; Starobinski, 1982; Guth and Pi, 1982).

The one drawback to the C-W inflationary scenario is that it requires the fine-tuning of the effective mass parameter of the theory in the de Sitter phase to a value close to zero (see, for example, Albrecht and Steinhardt, 1982b). It can be argued that such a model is special and that the setting of a parameter to zero is not so unnatural. However, since there is no unbroken symmetry that can result in such a finely tuned condition, such arguments are suspect. The C-W models provide a possible but unlikely support for the inflationary scenario.

In this paper I plan to outline a first attempt at finding a theory which leads to inflation without fine-tuning of parameters. The models I will discuss utilize the lesson learned from the C-W models -- inflation must occur in evolving stable phase -- but the inflation will occur for a wide and natural choice of all parameters.

Models which possess supersymmetry appear to be obvious candidates for at least two reasons. Firstly, supersymmetry is supposed to solve the unnatural hierarchy problem; shouldn't the same notion solve the unnatural inflation problem? Secondly, supersymmetric models are known to lead to radiative symmetry breaking without fine-tuning of parameters. (I use the term "radiative symmetry breaking" rather than "Coleman-Weinberg" because the latter term has come to be associated with potentials that are very flat for small values of the scalar field expectation value. Both terms refer to the breaking of symmetry through one-loop radiative corrections to the effective potential, but in the supersymmetry case this does not naturally lead to a potential that is flat near the region of small scalar field.)

The natural radiative symmetry breaking occurs especially in supersymmetry models with O'Raifeartaigh (O'R) symmetry breaking (O'Raifeartaigh, 1975). For example, consider a model with three left-handed chiral fields, A, X, Y with a superpotential:

$$W = \lambda_1 X A^2 + \lambda_2 Y \, (A^2 - M^2) \quad . \tag{1}$$

The scalar potential in tree approximation is given by:

$$\begin{aligned} V &\equiv \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 \equiv \sum_i \left|F_i\right|^2 \\ &= \lambda_1^2 \, |A|^4 + \lambda_2^2 \, |A^2 - M^2|^2 + |2\lambda_1 AX + 2\lambda_2 AY|^2 \quad , \end{aligned} \tag{2}$$

where i runs over the various fields and the same symbol has been used for the superfields and their scalar components. There is no state with V = 0 so supersymmetry is (O'R) spontaneously broken with a global minimum at $\langle A\rangle = \lambda_2 M/(\lambda_1^2 + \lambda_2^2)^{1/2}$. The value of $\langle X\rangle$ and $\langle Y\rangle$ remain undetermined at tree level, only constrained by

$$|X| = -\frac{\lambda_2}{\lambda_1} \langle Y\rangle \ . \tag{3}$$

Thus, the mass of the A field, given by $\sim 2\,\lambda_1\langle X\rangle$ (for large X), can be arbitrarily large. This kind of degeneracy is natural to O'R breaking since to break supersymmetry and have $V \neq 0$ at the minimum of the potential, there must be an algebraic inconsistency in setting all the F_i's to zero; to obtain inconsistency for one field expectation value (in this case the A field) at least one other field expectation value usually remains undetermined.

The degeneracy is broken by one-loop corrections to the effective potential - just the radiative symmetry breaking discovered by Coleman and Weinberg (1973):

$$V_{1\text{-loop}} = \sum_i \frac{(-1)^F}{64\pi^2} M_i^{\,4}(\phi) \, \ln \frac{M_i^{\,2}}{\mu^2} \tag{4}$$

where μ is a renormalization mass and $(-1)^F$ is +1 (-1) for bosons (fermions). For $X \gtrsim M$, the only X-dependent masses come from A and its supersymmetric partners. Their masses are such that the one-loop corrections are given by (Witten, 1981; Huq, 1976):

$$V_{1\text{-loop}} = \alpha M^4 \ln \frac{|X|}{\mu} \tag{5}$$

where α is a function of the scalar coupling constants. This form is very special because, unlike C-W GUT models, the only X-dependence appears inside the logarithm; this crucial behavior occurs because of special cancellations that occur in Eq. (4), as a remnant of the broken supersymmetry. The net result for the pure scalar theory is that the state with $X = 0$ is stabilized. For a gauge theory, the analog of Eq. (5) is (Witten, 1981)

$$V_{1\text{-loop}} = (\alpha' - \beta g^2) M^4 \ln \frac{|X|}{\mu} \tag{6}$$

where α' and β are functions of the scalar couplings and g is the gauge coupling constant. If $(\alpha' - \beta g^2) < 0$, the state with $\langle X \rangle = 0$ is destabilized compared to the state with large $\langle X \rangle$. (NOTE: The potential has only been determined for $X \gtrsim M$ for which it is independent of parameters; the shape of the potential for $X \lesssim M$ depends upon the choice of parameters and the $\langle X \rangle = 0$ state may be metastable or unstable). At large X ($\sim M e^{1/\alpha}$), the potential appears to become negative, which is impossible for a globally supersymmetric theory. In fact, asymptotic freedom forces g to decrease with large X and the sign of the logarithm term changes sign; thus V develops a stable minimum at this large value of X. Beginning with a single mass scale, M, we have generated a much larger mass scale. Witten (1981) hoped to be able to use this idea to generate the GUT scale from a theory in which the only weak scale has been introduced as a fundamental mass. Because usual hierarchy solutions attempt to generate the weak scale from a fundamental GUT scale, this sort of model has come to be known as a "reverse hierarchy" model.

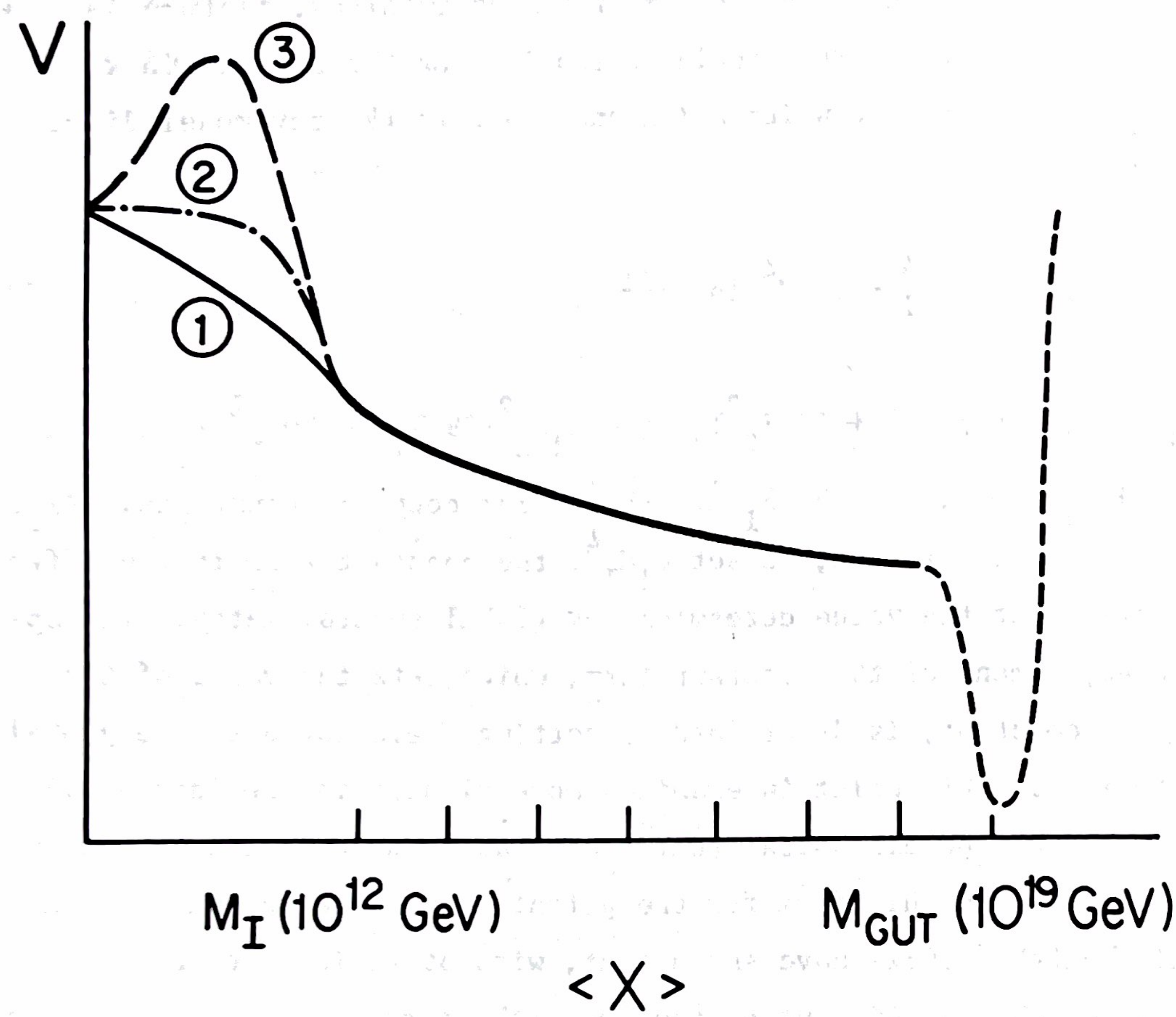
Dimopoulos and Raby (1982) attempted to extend this idea to produce both the weak and GUT scales through radiative corrections to a theory with only a single fundamental intermediate scale. In their model particles appear with masses of order $M_{GUT} = 10^{19}$ GeV $= M_I\, e^{1/\alpha}$; $M_W = M_I\, e^{-1/\alpha} = 10^5$ GeV; and $M_I = 10^{12}$ GeV. (The model is referred to as the "geometric hierarchy" model because $M_I^2 \approx M_{GUT} M_W$). The theory has many interesting properties that are discussed by Dimopoulos and Raby (1982). For the purpose of this paper, the most important feature is that the effective zero temperature scalar potential for the X field in the range $M_I \lesssim X \lesssim M_{GUT}$ can be written (in analogy with the toy model discussed above):

$$V_{eff} = c_1 M_I^4 - c_2 M_I^4 \ln \frac{|X|}{M_{GUT}} \quad (7)$$

where $c_1 = (\lambda_1^2 \lambda_2^2/(\lambda_1^2 + 30 \lambda_2^2)$, $c_2 = c_1\lambda_2^2(29 \lambda_1^2 - 50 g^2)/(80\pi^2(\lambda_2^2 + \lambda_1^2/30))$, and the λ_i are the scalar coupling constants. (For the purpose of illustration, we set $c_1 M_I^4$, the constant term in the effective potential, at the value determined by global supersymmetry. The appropriate adjustment of the constant term, which sets the value of the cosmological constant, is determined by setting V evaluated at the global minimum to zero. This point is examined more closely in the Addendum.) Once again it must be emphasized that there has been no special choice of parameters to obtain this form for the potential. (In a recent preprint, Hall & Hinchcliffe (1982) have shown that, without tuning the values of the couplings, the coefficient of the logarithm term in Eq. (7) according to the renormalization group equations changes sign when $X \sim 10^3 M_I$ or less for the simplest models, so that M_G/M_I is much less than supposed by Dimopoulos & Raby (1982). This tuning may be avoided by adding additional terms to the superpotential to change the renormalization group equations. In general, the effect they note can be made insignificant, if α is somewhat smaller (α~1/30) than they assume.) The shape of the potential is shown in Fig. 1, where nothing has been assumed for the shape of the potential for $X \gtrsim M$. We will assume that a transition takes place in at least some region (of size $\lesssim M_I^{-1}$) of the Universe which takes us from a state with X = 0 to a state with $X \lesssim M_I$; eventually, due to the potential, the state evolves to a value of $\langle X \rangle$ of order M_I and a value of $\langle X_I \rangle$ of order M_I^2; these will form the initial conditions for our analysis. The insensitivity of our results to the prehistory of the region is, of course, what makes this scenario natural and attractive.

I will now outline two means by which tremendous inflation can be produced through the subsequent evolution of the X field. These methods have been studied

Fig. 1. The zero-temperature effective potential for the geometric hierarchy model.



in collaboration with A. Albrecht, S. Dimopoulos, E Kolb, W. Fischler and S. Raby. A paper discussing our results associated with both methods will appear in a forthcoming publication (Albrecht, et al., 1982b).

Method I: The equation for the evolution of the X-field in the expanding Universe is given by:

$$\ddot{X} + 3H\dot{X} + V'(X) = 0 \tag{8}$$

where $V'(X) = -c_2 M_I^{\,4}/X$; H is the Hubble constant:

$$H^2 \simeq \frac{8\pi}{3}\frac{V(X)}{M_p^{\,2}}, \; M_p = \text{Planck mass} \simeq 10^{19} \text{ GeV}, \tag{9}$$

which gives rise to an effective damping term in Eq. (8). If one writes $V'(X) = -m^2(X)X$ where

$$m^2(X) = \frac{c_2 M_I^4}{X^2} , \tag{10}$$

the damping term dominates Eq. (8) provided that

$$4m^2(X) << 9H^2 \text{ or } X^2 >> \frac{c_2}{6\pi c_1} M_p^2 . \tag{11}$$

Suppose that the domination of the damping term occurs for some large value, $X = X_o$ which is $\approx .1 M_{GUT} = 10^{18}$ GeV. According to Eqs. (9, 10, 11), this requires $c_1 \gtrsim 50 c_2$. If we assume that $\lambda_1 \approx \lambda_2 \approx \lambda = .1$, we find from Eq. (6) that $c_1 > 27000 c_2$ (for g small) so that Eq. (11) can be satisfied without fine-tuning of parameters (see Addendum). If the damping term dominates, Eq. (8) is solved approximately by:

$$\dot{X} = c_2 M_I^4/3HX \tag{12}$$

or

$$\Delta t = \frac{3}{2} \frac{H}{c_2 M_I^4} \Delta X^2 . \tag{13}$$

Suppose that the evolution of X results in

$$\Delta X^2 \approx \frac{1}{e^2} M_{GUT}^2 - X_o^2 ,$$

where we assume Eq. (7) becomes unreliable for $M \sim M_{GUT}/e$ (the factor of 1/e is chosen just for the purpose of illustration; general choice of parameters is discussed by Albrecht, et al., 1982b). Then, the roll-over time, t_r, is given approximately by

$$\Delta t_r = \frac{3}{2} \frac{M_{GUT}^2}{e^2 c_2 M_I^4} H . \tag{14}$$

The scale parameter expands by a factor (Eqs. 9, 14):

$$\frac{R(t_r)}{R(t_o)} = \exp (H\Delta t_r) = \exp \frac{4\pi}{e^2} \frac{c_1}{c_2} \tag{15}$$

so that for $c_1 > 50\, c_2$, a fluctuation region of order M_I^{-1} $((10^{12}\text{GeV})^{-1} = 2 \times 10^{-26}\text{cm})$ expands to a size of order 10^{11} cm, much greater than the size of our observed Universe ($\sim 10^3$ cm) at the time of the completion of the transition. The inflation occurs near the very end of the logarithmic portion of the potential where $X \sim M_{GUT}$ and the degree of inflation is controlled by the ratio c_1/c_2. The fact that M_{GUT} is of order the Planck mass (Dimopoulos and Raby, 1982) in the geometric hierarchy model is unfortunate, but it is not required for the inflation to occur. Provided that c_1/c_2 is sufficiently large, the value of M_{GUT} can be reduced to $10^{-2}\, M_{Planck}$ where quantum gravitational effects are not bothersome. Thus, as a generic potential, Eq. (7) results in inflation without any special tuning of the parameters of the theory and without any detailed dependence on the shape of the potential for $X \lesssim M_I$.

Method II: For the second method, the inflation depends on radiation damping due to the particle production resulting from the time variation of the X field. In Albrecht, et al. (1982a), the time variation of the Higgs field was studied in relation to the problem of reheating at the end of the inflationary scenario. Suppose, as in that case, one postulates that the time varying field radiates an energy density per unit time given by

$$\delta \approx a(\dot{X})^d\, X^{5-2d} \tag{16}$$

where d is arbitrarily chosen. As a first test, suppose that the X field in our case has couplings such that $\delta = a\, X\, \dot{X}^2$; the reasonableness of this guess will be discussed at the end of the analysis. The equation of motion for the X field is given by

$$\ddot{X} + 3H\dot{X} + aX\dot{X} + V'(X) = 0 \tag{17}$$

where the third term is the effect due to δ. For $X \gtrsim M_I$, the δ-term dominates the time evolution equation. The approximate solution to Eq. (17) is given by

$$\dot{X} = c_2\, M_I^4/aX^2 \tag{18}$$

or

$$\Delta t = \frac{a}{3c_2\, M_I^4}\, \Delta X^3 \quad . \tag{19}$$

If X rolls from $X \sim M_I$ to $X \sim M_{GUT}$, the rollover time is given by

$$\Delta t_r = a\, M_{GUT}^3 / c_2\, M_I^4 \, . \tag{20}$$

If the Hubble constant, H, is given by

$$H^2 \simeq \frac{8\pi}{3} \frac{c_1 M_I^4}{M_P^2} \tag{21}$$

we find that the scale factor after the inflation is given by

$$\frac{R(t_r)}{R(t_o)} = \exp(H\Delta t_r) = \exp\left[\frac{8\pi\, c_1}{3} \frac{a\, M_{GUT}^3}{c_2\, M_p\, M_I^2}\right] > e^{10^{12}} \; ! \tag{22}$$

The resulting expansion is enormous, more than enough to solve the cosmological puzzles if we begin with a region with an initial size $\sim M_I^{-1}$.

Two major warnings should be given concerning Method II, however. Firstly, the results depend crucially on d in Eq. (16) having a value $1 < d < 2.5$; this was not the case for the studies of Albrecht et al., (1982a). The assumption is only reasonable if X couples in a special way to light (nearly massless) particles. Secondly, it is unlikely that the X particle couples strongly to any light particles. Dimopoulos and Raby have especially designed the model to ensure that X decouples from light fields as it evolves in order to maintain their weak scale masses. Even in a more general theory, we expect the particles that couple to X to have masses of order $\langle X \rangle$, which becomes very large. Only in a very special model could Method II be possible, but the huge inflation effect that results makes it worth searching for such a model. Perhaps supersymmetry can permit the X to couple to fields which remain light due to cancellations coming from coupling to other fields with growing expectation values?

After the inflation must come the reheating, and here may lie the Achilles heel of this scenario. As X gets large, asymptotic freedom leads to the decrease of the gauge coupling constant until the sign of the logarithmic potential in Eq. (7) changes, producing a stable minimum for X. If the curvature of the effective potential (and the mass of the X, M_X) remains small ($\sim \frac{M_I^2}{M_P}$), X reaches the minimum and stops (due to the damping term); $\dot{X}$ remains small, so no reheating occurs. However, when X becomes large, two other effects become important: (a) perturbation theory breaks down and (b) gravitational corrections become important. The breakdown of perturbation is not likely to change the curvature near the minimum. The minimum occurs where the coefficient of the logarithm changes sign at which point the values of g and λ are still small. The renormalization group, which should still be reliable at the minimum, suggests that the curvature at the minimum is only corrected by logarithmic factors, and so remains too small for significant reheating. The gravitational effects can be estimated assuming one wishes to imbed this model in a locally supersymmetric model (Weinberg, 1982). (I will assume that the Planck mass, M_P, is treated as a fundamental parameter rather than a dynamically generated parameter or else the gravitational corrections would be scaled by <X> and become important for all X. In order to ensure that gravitational effects become important only for large X, M_P must be fixed by hand.) It appears from crude computations a model might be adjusted so that there is sufficient inflation due to the logarithmic tree potential, but also sufficient reheating due to gravitational effects to a temperature $\gtrsim 10^{-3} M_I$ (see Addendum for discussion). (Here the constant term, $c_1 M_I^4$, that appears in Eq. (7) is generated dynamically as the difference in energy density between the potential at $X \sim M_I$ and the minimum $X \sim M_P$ (where our Universe now exists with a cosmological constant of zero).) The details of our study will be presented in Albrecht, et al. (1982b).

Reheating is crucial because (a) the thermal energy must be recovered so that the entropy of the Universe rises to a large value to correspond with observation, and (b) baryon asymmetry must be generated <u>after</u> the inflation. Since there exist color triplet Higgs mesons in the theory which have masses $\sim M_I$, if the Universe can recover to a temperature even as small as $\sim 10^{-3}$ M, it might be sufficient to ensure that the usual sort of baryon asymmetry scenarios (in which light mesons begin from near-equilibrium and then fall out of equilibrium (Dimopoulos and Raby, 1982; Albrecht, et al., 1982b) succeeds.

Much work is required to check whether the reheating can be accommodated with the rest of the features of the geometric hierarchy model, but if it is possible <u>all the successes of C-W inflation are obtained without any fine-tuning of parameters.</u> In fact, it appears that the problem of producing the inhomogeneities that grow into galaxies is more easily solved in models for which the inflation occurs when the scalar expectation value is large compared to H (as occurs naturally in this model, but not in the C-W models). The perturbation $\delta\rho/\rho$, evaluated when a scale enters its horizon has been shown (e.g. Bardeen et al., 1982) to be proportional to the value of $\Delta\phi/\bar{\phi}$ during the inflation epoch, where $\Delta\phi$ is the fluctuation about the mean value, $\bar{\phi}$. In C-W inflation, $\Delta\phi \sim H$ and $\bar{\phi} < H$, so $\Delta\phi/\bar{\phi}$ is large; as a result $\delta\rho/\rho$ is found to be too large compared to the value required by astrophysics ($\sim 10^{-4}$) to explain galaxy formation without producing a large background microwave anisotropy. In the supersymmetry model, $\Delta X \sim H << X$ during inflation and one can adjust parameters so that $\delta\rho/\rho \sim 10^{-4}$. Furthermore, the de Sitter scalar field fluctuations ($\propto H^2$) which can cause problems with C-W inflation scenarios (Vilenkin and Ford, 1982; Linde, 1982b) can be shown to be insignificant in this case, again because X is large compared to H in the inflationary phase (Albrecht et al. 1982b; Vilenkin, private communication).

As an amusement, it is interesting to speculate how the behavior of the effective potential for $X \leq M_I$ can affect the inflationary cosmology. The three possibilities are shown in Fig. 1, corresponding to (1) negative, (2) zero (analog of C-W) or (3) positive curvature of the potential near $X = 0$. As has been emphasized, which case is obtained depends upon the choice of parameters and inflation occurs in any case; therefore, which case one obtains has no effect on the predictions for scales of the size of our observable Universe. However, there can be different predictions for scales much larger (by a factor of 10^{30} or more) than the size of our observed Universe. Since such predictions are <u>in principle</u> unable to be tested by any human (protonic) observer, I call this branch of cosmology "metaphysical cosmology" or meta-cosmology for short. Each of the three cases shown in Fig. 1 can lead to a distinctive meta-cosmology:

(1) <u>Spinodal Metacosmology</u> - The potential has negative curvature at $X = 0$. At some finite temperature, the $X = 0$ phase becomes unstable and fluctuations drive different regions towards different SSB minima; the phenomenon of dividing the whole Universe (no region remains in the metastable phase) into different developing phase regions - fluctuation regions - is analogous to spinodal decomposition in condensed matter physics (Steinhardt, 1982). The different SSB minima are related by continuous or discrete symmetries. When different regions come together for which the phases contained therein differ by a discrete (continuous) symmetry, one expects domain walls (monopoles) to be formed. Of course the inflation pushes these topological defects to distances beyond which we can ever hope

to observe and before we could even reach them, the domain walls collapse gravitationally (Hawking, private communication). We conclude for the Spinodal Universe, that even though the Universe near us is very nearly homogeneous and isotropic, the Universe far away from us (near the edges of fluctuation regions) might be quite different. This is a startling possibility from a philosophical point of view.

(2) Single Universe Metacosmology: With special fine-tuning of parameters, the effective potential for the supersymmetry case can be made to be flat (in the C-W GUT sense, curvature $\lesssim H^2$) near the origin. In this case, the Hawking-Moss (1982) analysis suggests that the universe rolls all at once into the same SSB phase. There are no monopoles or domain walls formed as topological defects in the transition because there is only one "fluctuation region". The total Universe is uniform, homogeneous and isotropic. This metacosmology yields the simplest picture of our Universe.

(3) Regenerative Meta-Cosmology: If parameters are chosen so that there remains a barrier with a curvature $\gtrsim H^2$ at $X \approx 0$, the $X = 0$ phase remains metastable even as T approaches zero. Because $Z = 0$ is metastable, rather than stable, quantum fluctuations of the type studied by Coleman and Deluccia (1980) - analogs of flat space bubbles - are produced rarely in the metastable Universe. Their interiors contain evolving stable phase. These bubbles occupy only a small fraction of the total Universe, most of which remains in the metastable phase and continues to expand forever. The expectation value of X begins $\lesssim M_I$ and evolves along the potential. The resulting bubble inflates to a size much larger than our observed Universe. The interior is perfectly Robertson-Walker. Observers inside the bubble can never observe the bubble wall because the bubble expands at nearly the speed of light. Bubbles are produced so rarely that they almost never collide. When a new bubble is formed, it regenerates a new Universe which can never contact our own. New Universes are regenerated forever because the Universe is only percolated (fractally) as time approaches infinity. (See Albrecht, et al., 1982b for more elaborate discussions.) The regenerative scenario is probably the most surprising and radical cosmological possibility.

Linde (1982c) has noted (in response to this talk) that a regenerative metacosmology can avoid the usual problem of the initial cosmological singularity! In almost any other cosmology which has an early epoch in which the Universe was very hot, extrapolating backwards in time leads to an unavoidable space-time singularity; this naturally makes physicists uncomfortable. In the regenerative metacosmology the Universe could begin ($t = 0$) in the metastable phase at $T = 0$ (actually $T_{Hawking}$); The metastable region never has to be hot so there need never be an initial singularity. The Universe even today could be mainly in this phase, with only a few rare bubbles in which true thermal matter and ourselves can exist.

Why should the Universe have begun in such a metastable phase? Perhaps because the $X = 0$ phase is a state of highest symmetry. It should also be noted that there remains a problem if time is continued for $t < 0$. A full de Sitter spacetime has a contraction phase for $t < 0$ and an expansion phase for $t > 0$. If the Universe began in a metastable symmetric phase with $T = 0$ and $t = -\infty$, it would complete the transition before $t = 0$ and there would be no inflation phase $t > 0$. If one insists upon only beginning at $t = 0$, the singularity problem is only solved somewhat artificially (Linde, private communication).

What is not emphasized in Linde's paper, however, is that the CW GUTs inflation scenario leads, more likely, to a spinodal metacosmology because the barrier has a height $\ll H^4$ as $T \to 0$. On the other hand, in a model like the one considered in this paper, where the inflation does not depend on the properties of the potential for small values of the order parameter, a high barrier and, thus, the regenerative metacosmology clearly occur for a wide range of parameters. Thus, it appears possible that one more fundamental mystery of early cosmology might, in principle, be explained with an appropriate inflationary Universe.

This sort of amusing speculation only serves to emphasize the importance of no fine-tuning in the inflationary scenario. The "geometric hierarchy" model under consideration may not be a correct model for particle physics, but it illustrates how inflation can be achieved without unnatural assumptions. The "tricks" used for this case, Methods I and II, may well be useful for a more realistic theory not yet discovered. By increasing our catalog of techniques for accomplishing inflation, we can hope to raise the inflationary Universe hypothesis from the level of Possible to the level of Likely.

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REFERENCES

Abbott, L., Farhi, E. & Wise, M.B., (1982). Particle Production in the New Inflationary Universe. Phys.Lett. 117B, 29.

Albrecht, A. & Steinhardt, P., (1982a). Phys. Rev. Lett. 48, 1220.

Albrecht, A. & Steinhardt, P., (1982b). Coleman-Weinberg Models and the Inflationary Universe. Penn preprint, to be published.

Albrecht, A., Steinhardt, P.J., Turner, M.S. & Wilczek, F., (1982a) Phys. Rev. Lett. 48, 1437.

Albrecht, A., Dimopoulos, S., Fischler, W., Kolb, E., Raby, S. & Steinhardt, P.J., (1982b). Proceedings of the Third Marcel Grossman Conference, to be published.

Bardeen, J., Steinhardt, P.J. & Turner, M.S., (1982). Spontaneous Creation of Almost Scale-Free Density Perturbations in an Inflationary Universe. Penn preprint UPR-2020T.

Coleman, S. & DeLuccia, R., (1980). Phys. Rev. D21, 3305.

Coleman, S. & Weinberg, E., (1973). Phys. Rev. D7, 788.

Dimopoulos, S. & Raby, S. (1982). "Geometric Hierarchy", Los Alamos preprint, LA-UR-82-1282, to be published.

Dolgov, A. & Linde, A., (1982). Baryon Asymmetry in Inflationary Universe. Phys. Lett. 116B, 329.

Guth, A., (1981). Phys. Rev. D23, 347.

Guth, A. & Pi, S.-Y., (1982). Phys. Rev. Lett. 49, 1110.

Guth, A. & Weinberg, E., (1982). Could the Universe have Recovered from a Slow First-Order Phase Transition, to be published in Nucl. Phys. B212, 321.

Hall, L. & Hinchliffe, I., (1982). Phys. Lett. 119B, 128.

Hawking, S., (1982). The Development of Irregularities in a Single Bubble Inflationary Universe. Phys. Lett. 115B, 295.

Hawking, S. & Moss, I., (1982). Phys. Lett. 110B, 35.

Huq, M., (1976). Phys. Rev. D14, 3548.

Linde, A., (1982a). Phys. Lett. 108B, 389.

Linde, A., (1982b). Scalar Field Fluctuations in Expanding Universe and the New Inflationary Universe. Phys. Lett. 116B, 335.

Linde, A., (1982c). Non-singular Regenerating Inflationary Universe, unpublished, Lebedev preprint.

O'Raifeartaigh, L., (1975). Nucl. Phys. B96, 331.

Starobinski, A., (1982). Talk given at the Nuffield Workshop on the Very Early Universe.

Steinhardt, P., (1982). A New Cosmology. Proceedings of the Second Moriond Astrophysics Workshop, ed. P. Crane, T. Gaisser, D. Hegyi and J. Van Tranh.

Vilenkin, A. & Ford, L., (1982). Gravitational Effects upon Cosmological Phase Transitions. Phys. Rev. D26, 1231.

Weinberg, S., (1982). to be published.

Witten, E., (1981). Phys. Lett. 105B, 267.

ADDENDUM

Eq. (7) only represents an initial guess for the form of the effective potential based solely on an analysis of global supersymmetry. The constant term that is computed ensures that the potential, V, is greater than zero at the minimum, as required by supersymmetry. The analysis began by using this constant as a rough estimate for setting the cosmological constant in the equations of motion for the evolution of the Universe. Without some concrete calculation to determine the effective potential near $\langle X \rangle = M_{GUT}$, the constant cannot be adjusted so as to make the cosmological constant at the minimum of the potential equal to zero.

As an improvement on this guess we (Albrecht, et al., 1982b) have analyzed the lowest order gravitational corrections to the effective potential. Near $\langle X \rangle = M_{GUT} \sim M_p$, higher order gravitational corrections certainly become important, but we have considered the lowest order corrections only to obtain a qualitative feeling of what to expect. We have also assumed that the Planck scale is introduced as a fundamental scale and not generated dynamically. If it were generated dynamically by the expectation value of X, gravitational corrections would be incalculable and important for all X. By introducing the Planck scale as a fundamental scale we must accept the fact that we have, to some degree, violated the spirit of the reverse hierarchy scheme, but at least we can begin to calculate.

Instead of Eq. (2), V with first order gravitational corrections takes the form (Weinberg, 1982):

$$V = \exp(8\pi GD)\left[\sum_{i,j} h_{ij} F_i F_j^* - 24\pi G|W|^2\right]$$

$$F_i = \frac{\partial W}{\partial \phi_i} + 8\pi GW\left(\frac{\partial D}{\partial \phi_i}\right) , \qquad (23)$$

where D is the supersymmetry term derived from the gauge interactions:

$$D = -g([A^*,A] + [X^*,X] + \text{irrelevant terms}) \qquad (24)$$

Inserting Eqs. (1,24) into Eq. (23), we find that the corrections are given by:

$$\Delta V_g \sim c_3 \frac{M_I^4}{M_p^4}(X^4 - c_4 M_p^2 X^2) + \Lambda , \qquad (25)$$

where c_3 and c_4 are constants that we cannot accurately compute.

Consider the effects of adding V_g to the effective potential, Eq. (6). First, note that V_g becomes important compared to the term logarithmic in X when $X \sim M_p$; for large X the correction leads to a minimum near $X \sim M_p$ with a curvature that is generally greater than that obtained from the logarithmic term alone. The constant, Λ, is chosen so as to set the cosmological constant at the minimum of the potential (corresponding to our present Universe) equal to zero. We have computed numerically the roll-over of the X field for the combined potential and we have discovered that for a wide (and natural) range of parameters sufficient inflation can be achieved due simply to the effects of the $3H\dot{X}$ term in the equation of motion, just the same effect as Method I used for Eq. (7). To analyze the reheating problem (which is aided by the potential which is steeper near the minimum) we have considered various forms for δ:

$$\delta = aX^5\left(\frac{\dot{X}}{X^2}\right)^a\left(\frac{M_I}{X}\right)^b \qquad (26)$$

We find that for a = 2, $b \le 2$ reheating can be achieved for a wide range of parameters; for a = 2 and $b \ge 2$, almost no reheating occurs. This is an improvement on the situation in the absence of gravitational effects where none of these choices resulted in reheating. We have also been rather conservative in our allowed choices of the parameters, and a more precise calculation of gravitational effects could help matters. As it stands, we are presently examining which is the appropriate choice for δ for the supersymmetric model. For the Dimopoulos-Raby model it appears that because of the decoupling of X from light fields in that model, $b \ge 6$. However, it is not yet clear whether a variation on this class of models might not yield a case where $b \le 2$ is appropriate. More details will be provided in Albrecht, et al. (1982b).