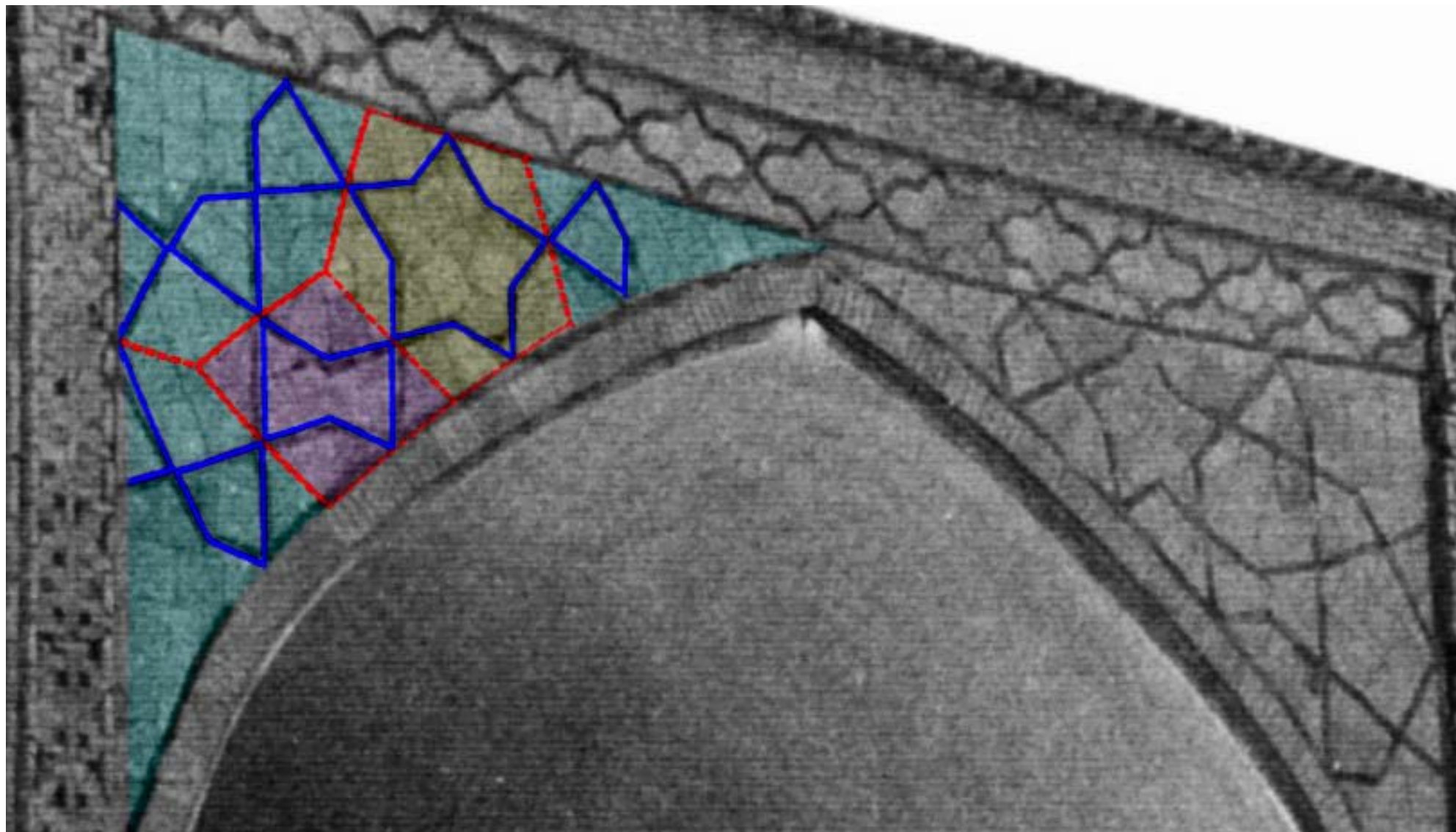
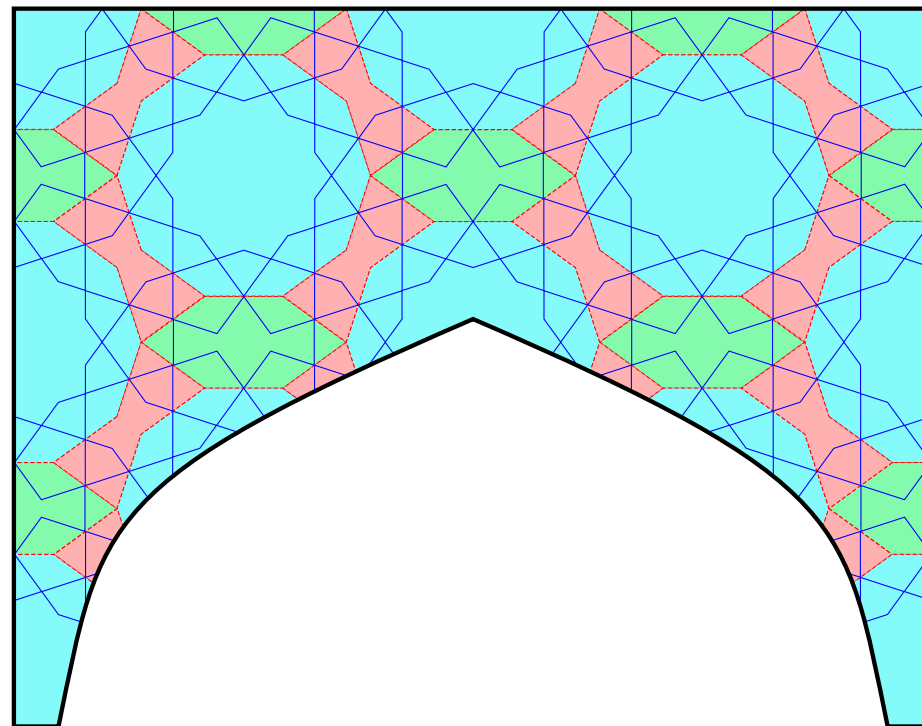
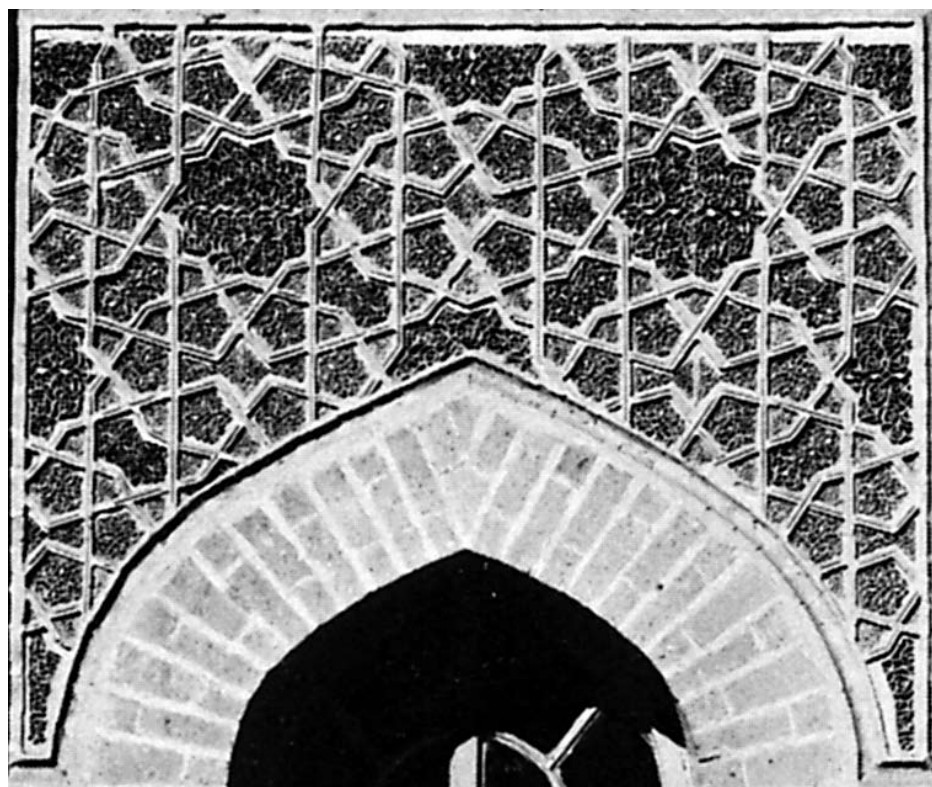


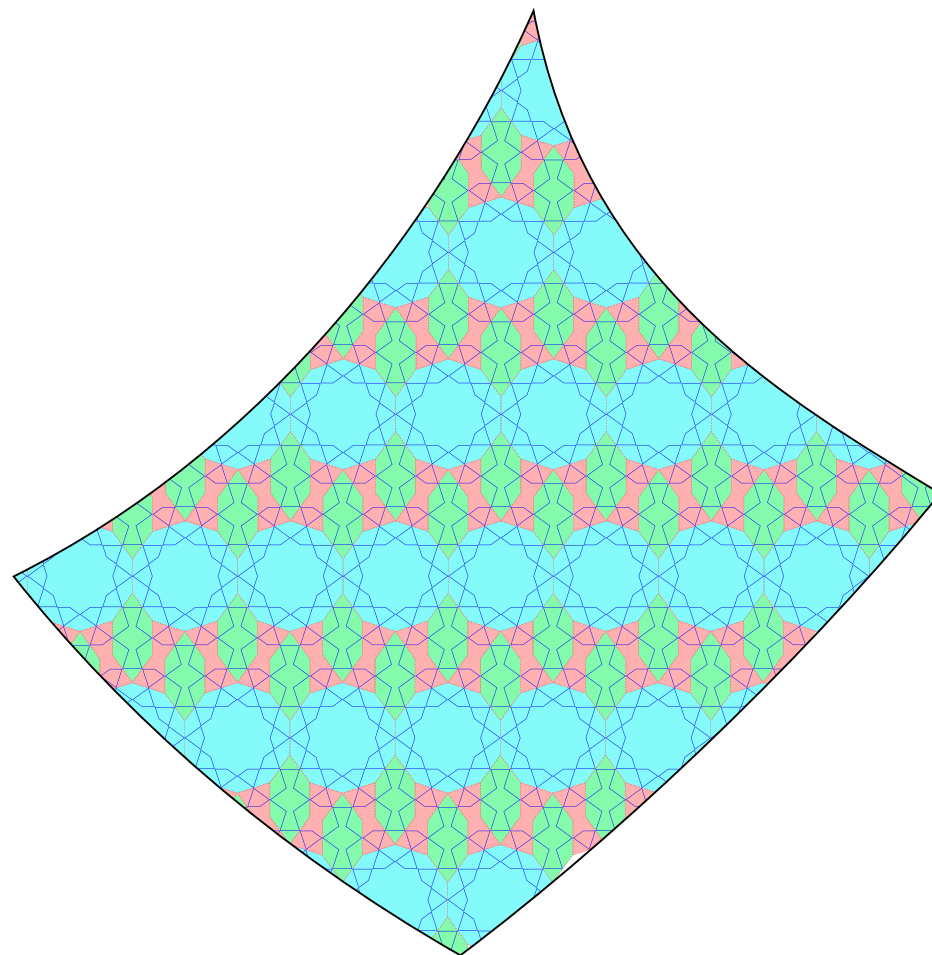
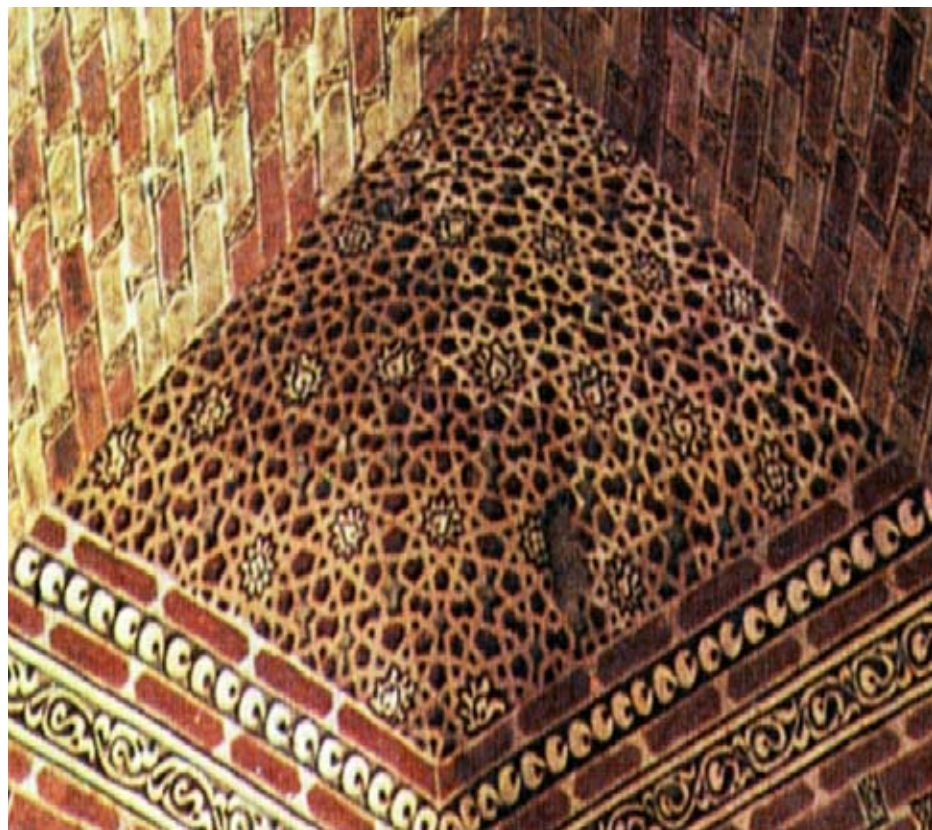
Supplementary Figure S1 Examples of tilings based on the same underlying geometry as the pattern on the Timurid shrine of Khwaja Abdullah Ansari at Gazargah in Herat, Afghanistan illustrated in Fig. 1E. Unit cell (pattern Fig. 1D) marked by a yellow rectangle. **(A)** the Seljuk Congregational Mosque in Ardistan, Iran (c. 1160 AD). **(B)** The Timurid Tuman Aqa Mausoleum in the Shah-i Zinda complex in Samarkand, Uzbekistan (1405 AD). **(C)** The Safavid Darb-i Kushk shrine in Isfahan, Iran (1496 AD). **(D)** The Mughal I'timad al-Daula Mausoleum in Agra, India (c. 1622 AD). [See note 10 for references.]



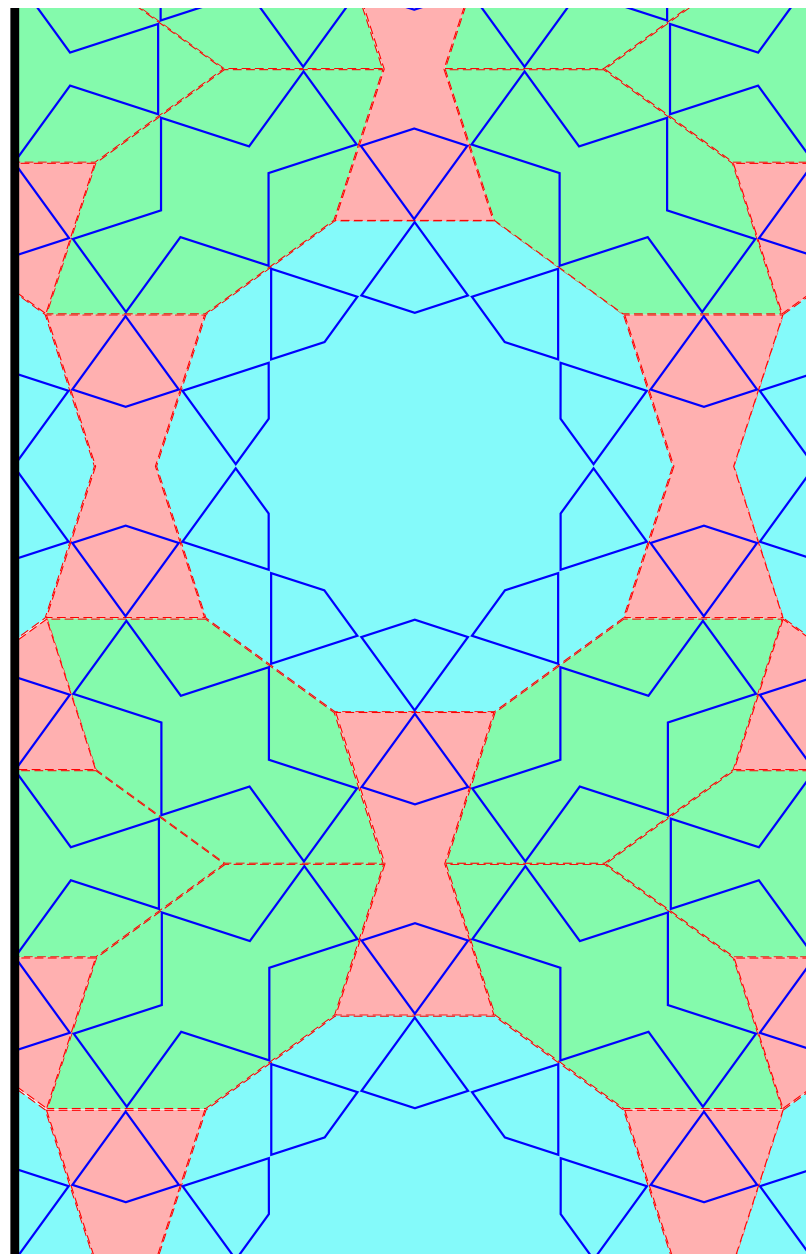
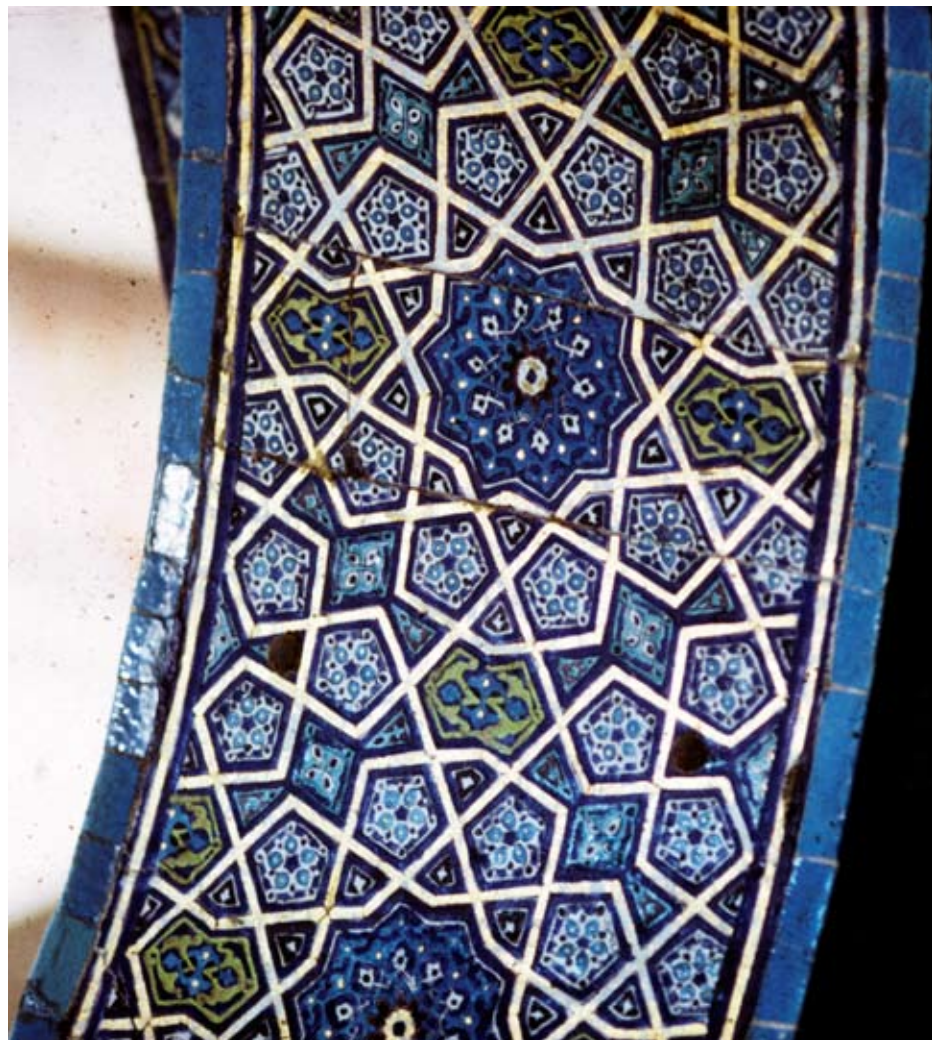
Supplementary Figure S2 Photograph of the spandrels of the sanctuary iwan from the Seljuk Great Mosque in Nayriz, Iran (pattern c. 15th century), showing the rhombus (purple) and pentagon (yellow) girih tiles. [From ref. 11 in the text.]



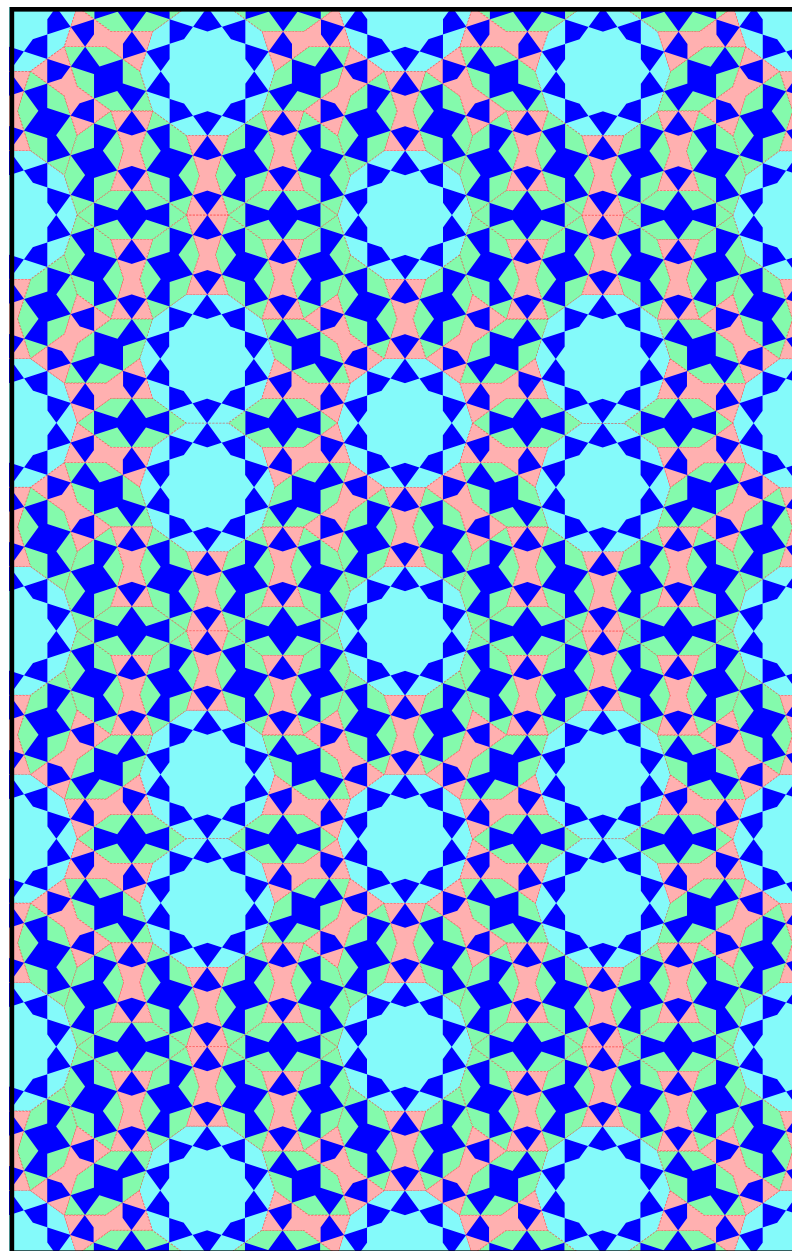
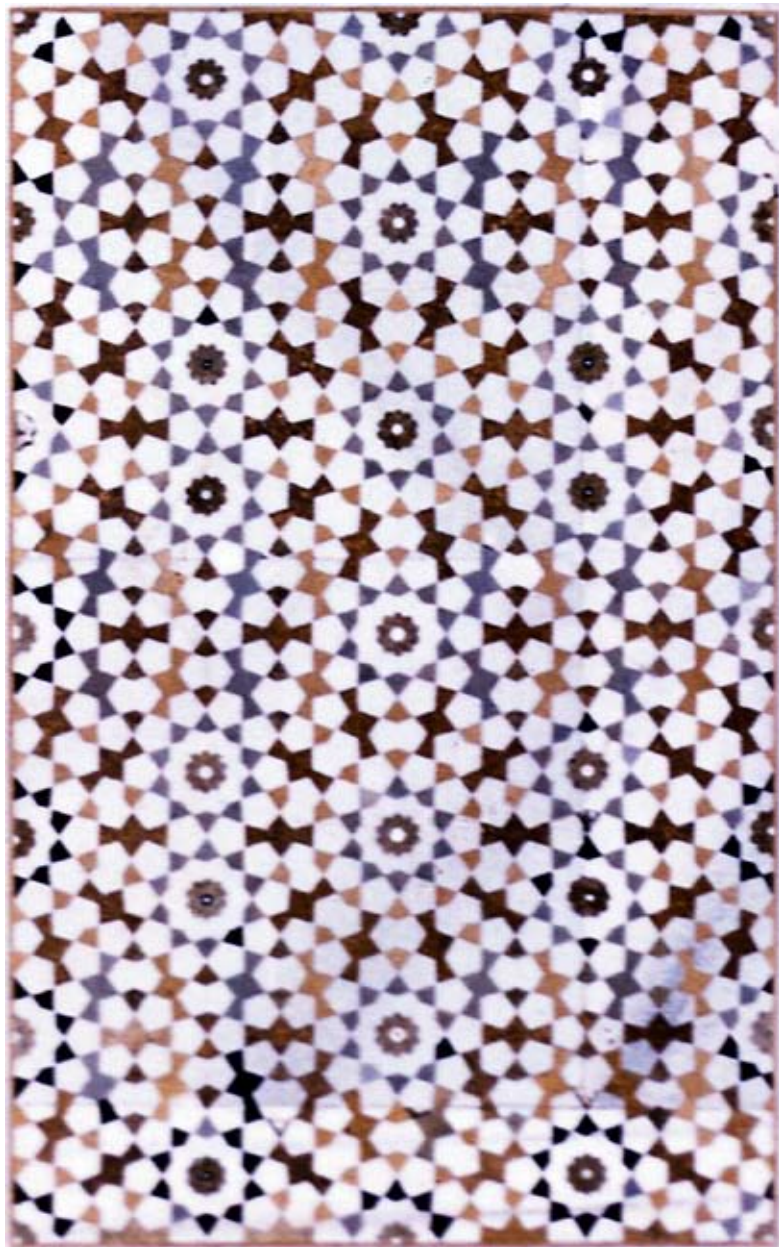
Supplementary Figure S3A Spandrel from the Abbasid Al-Mustansiriyya Madrasa in Baghdad, Iraq (1227-34 AD), and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 12 for reference.]



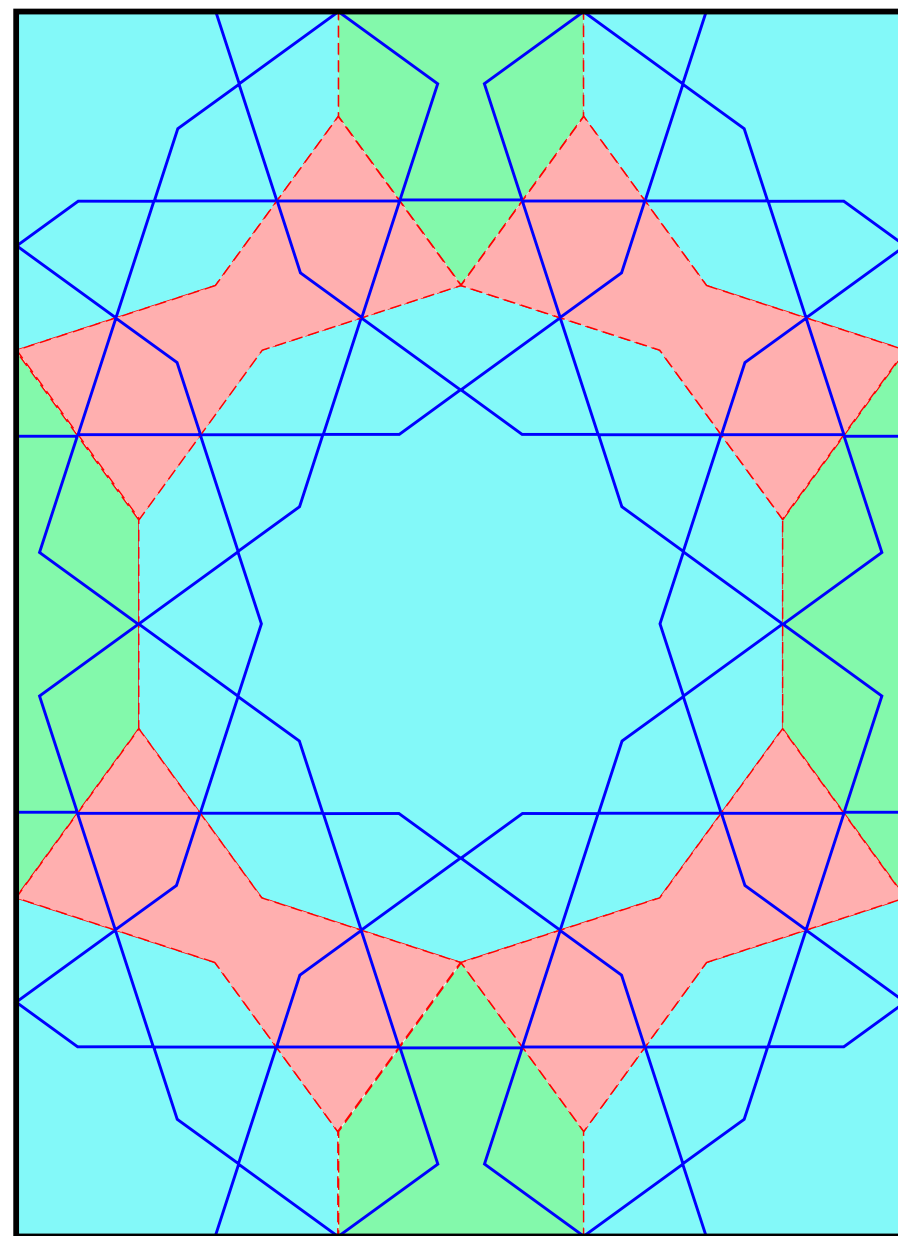
Supplementary Figure S3B Interior ceiling on a vault in the gallery of the Ilkhanid Uljaytu Mausoleum in Sultaniya, Iran (1304 AD), and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 12 for reference.]



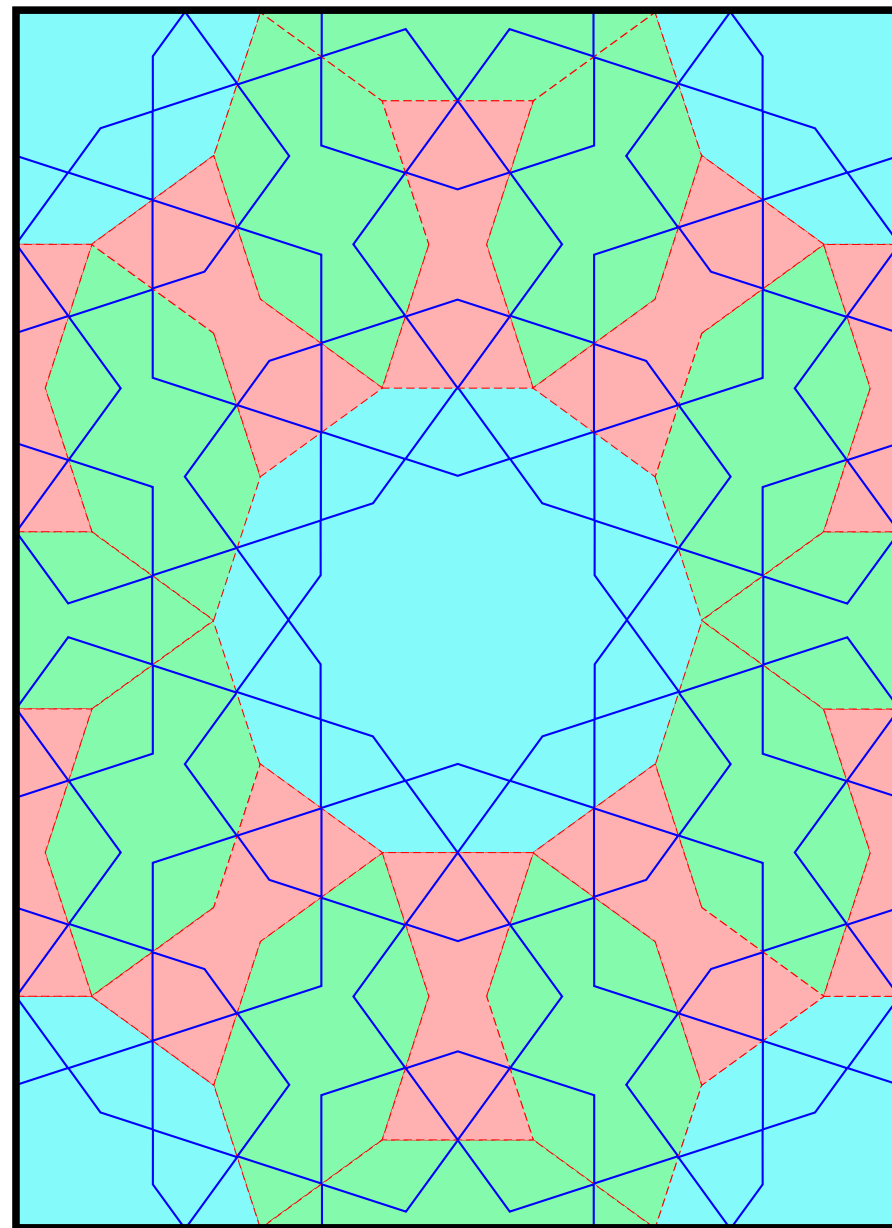
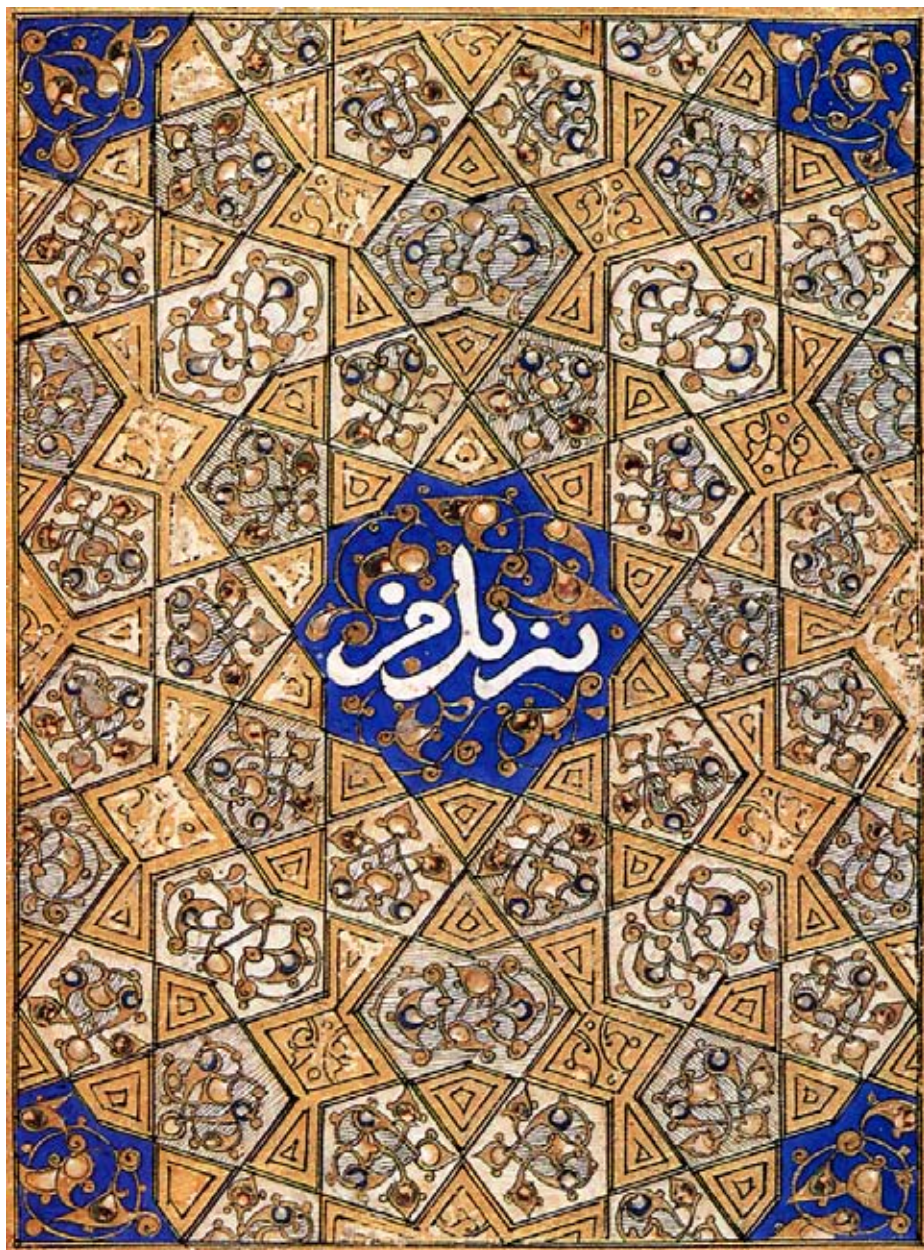
Supplementary Figure S3C Interior archway at the opening of the Sultan's Lodge in the Ottoman Green Mosque (Yesil Cami) in Bursa, Turkey (1424 AD), and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 12 for reference.]



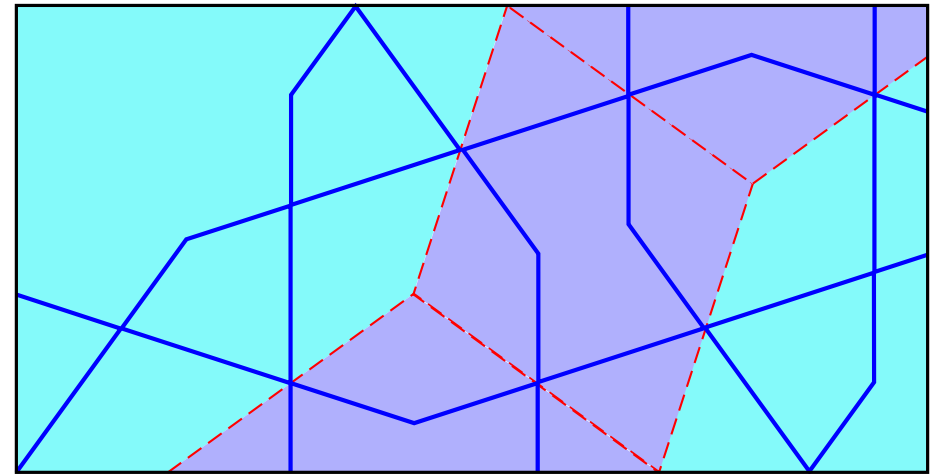
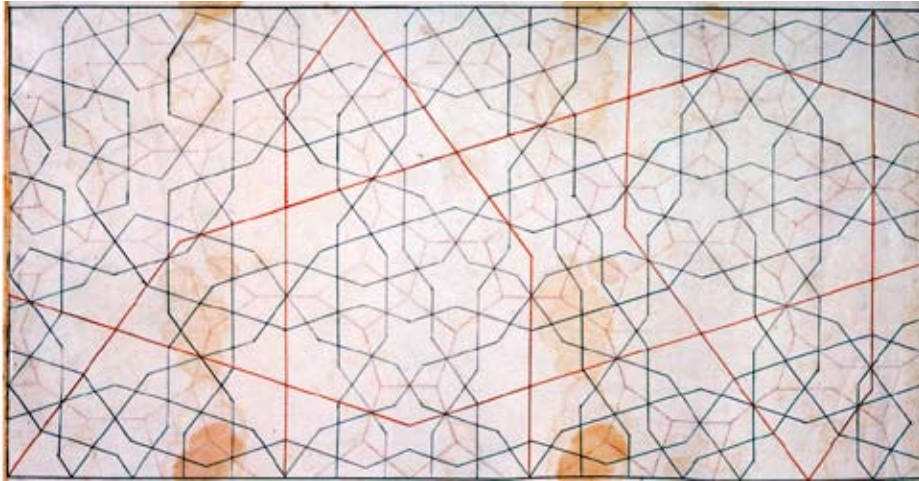
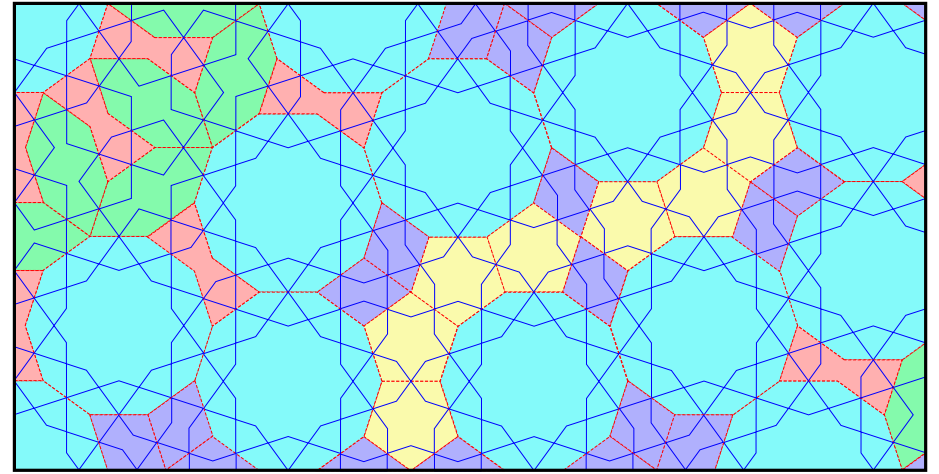
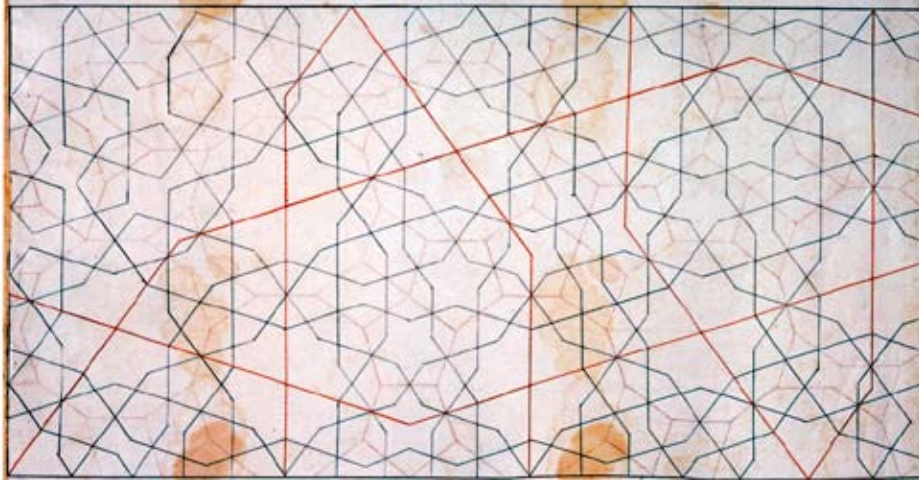
Supplementary Figure S3D External panel from the Mughal I'timad al-Daula Mausoleum in Agra, India (c. 1622 AD), and reconstruction of the pattern using the girih tiles, where the blue line decoration of Fig. 1F has been filled in to form a solid blue region. [See note 12 for reference.]



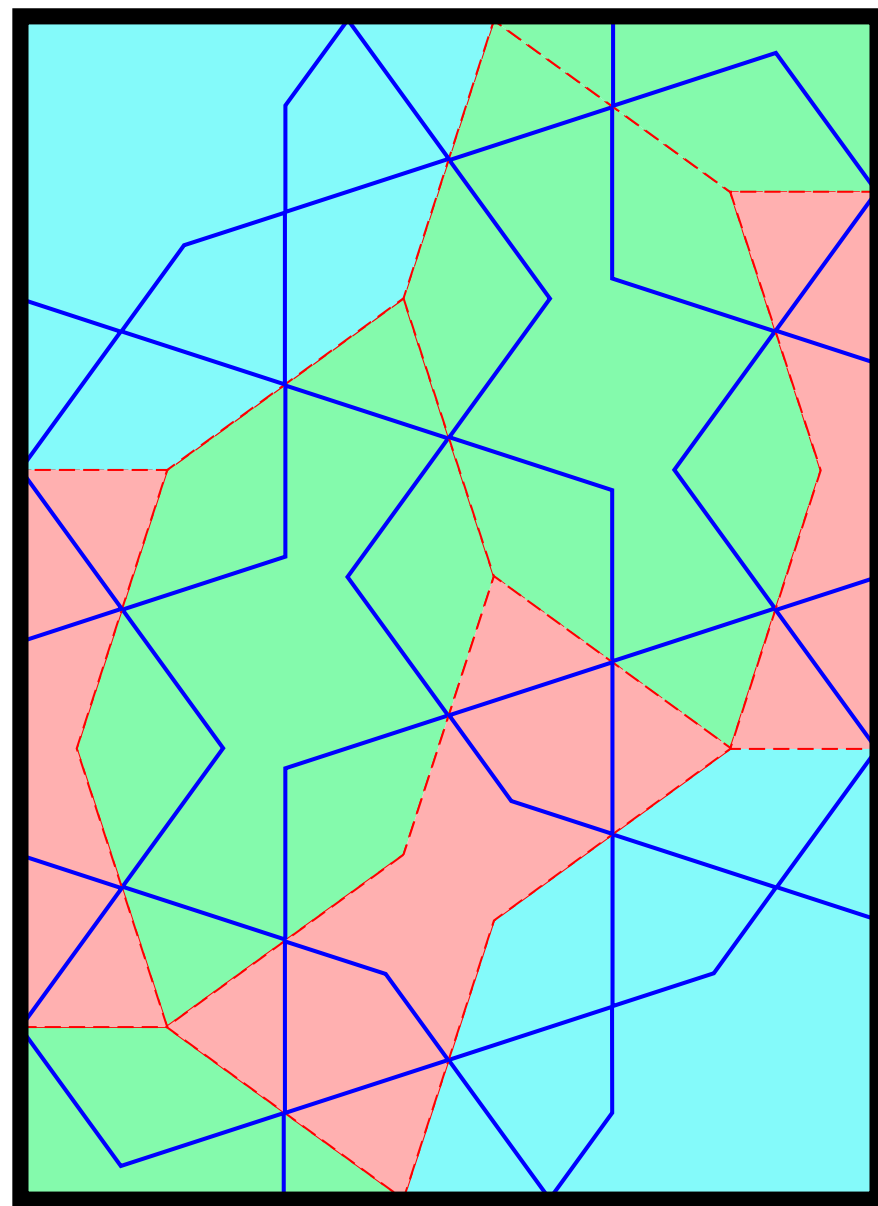
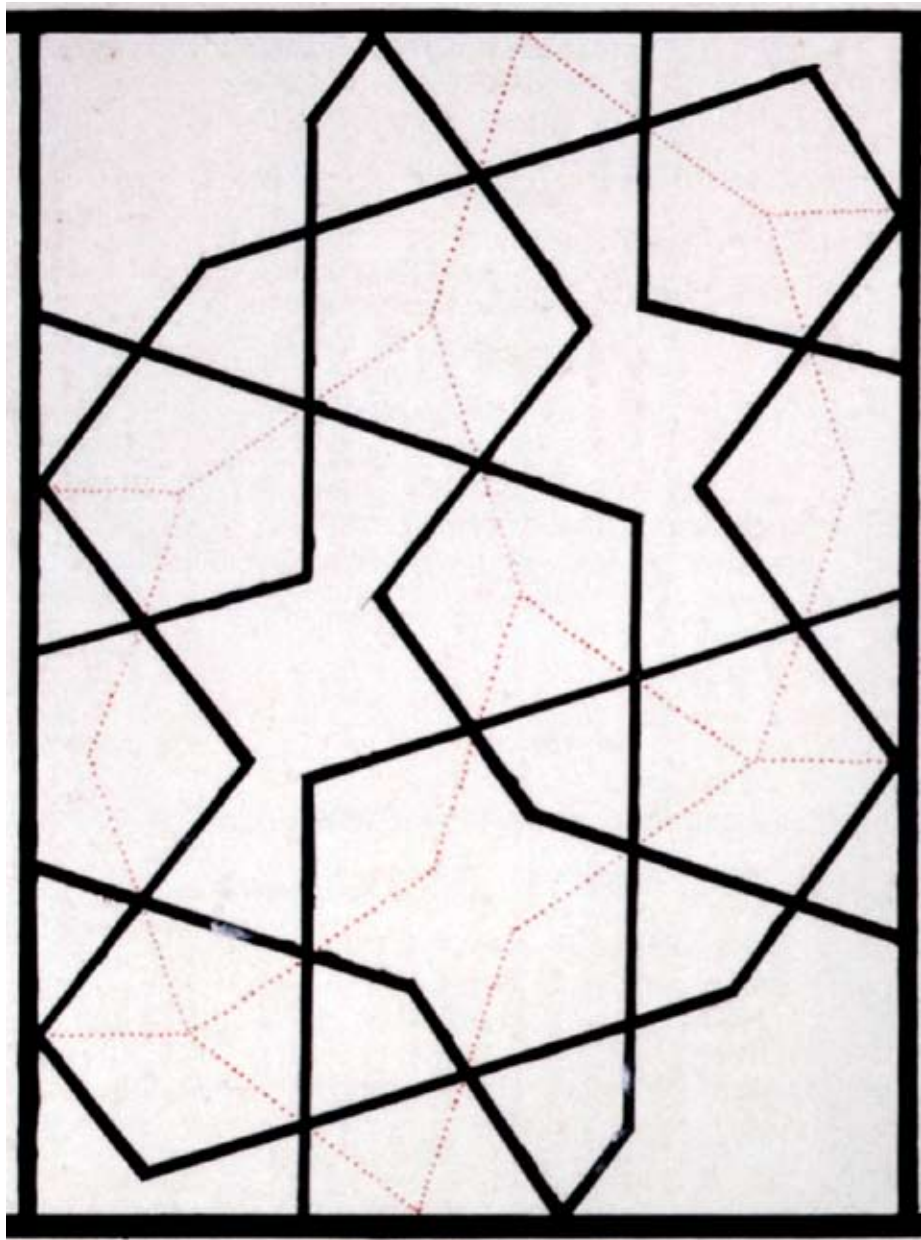
Supplementary Figure S3E Mamluk Quran of Sandal (1306-15 AD), and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 12 for reference.]



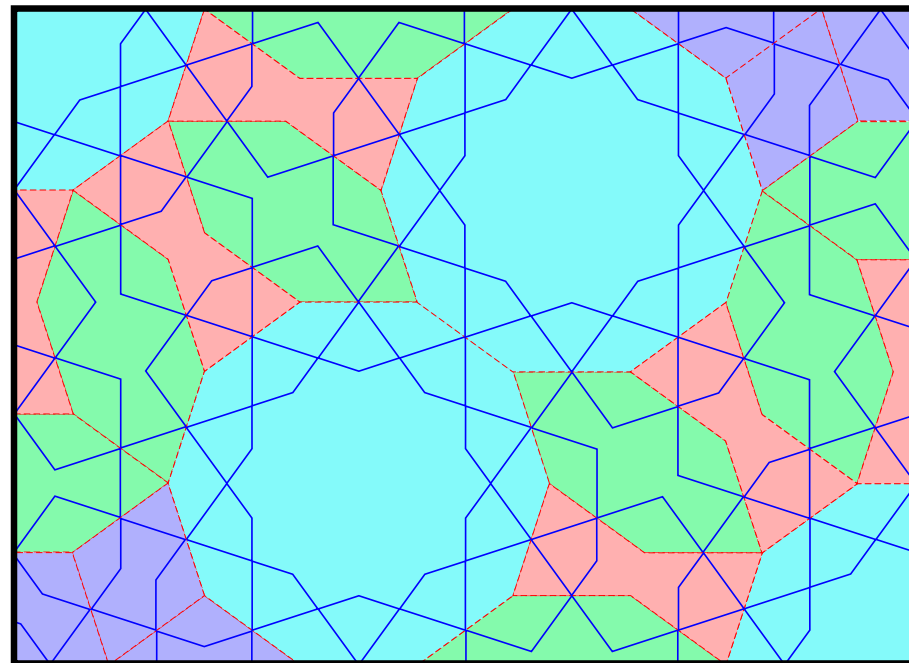
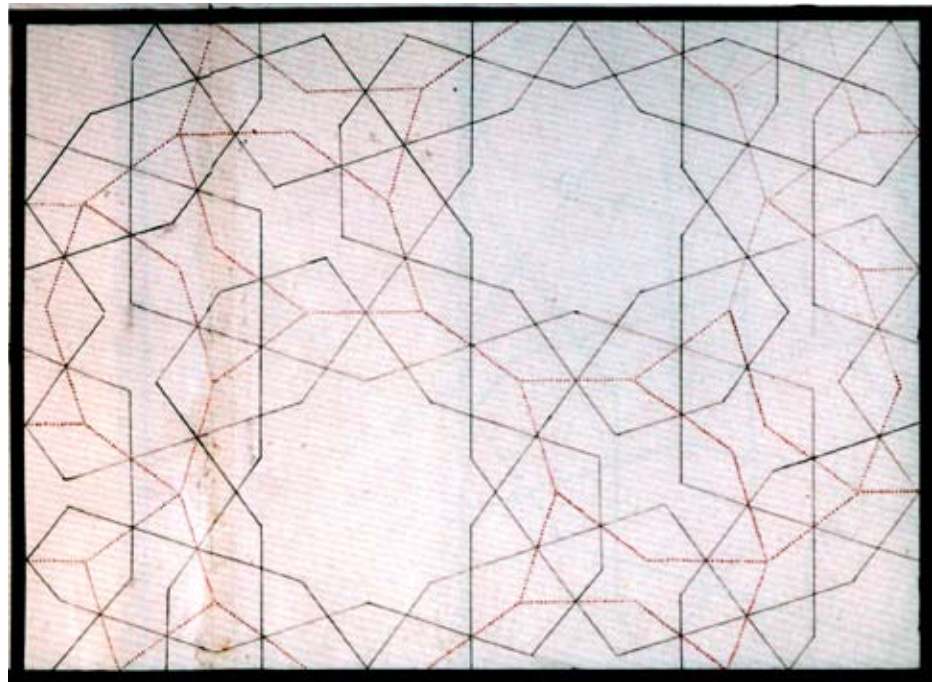
Supplementary Figure S3F Mamluk Quran of Aydughdi ibn Abdallah al-Badri (1313 AD), and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 12 for reference.]



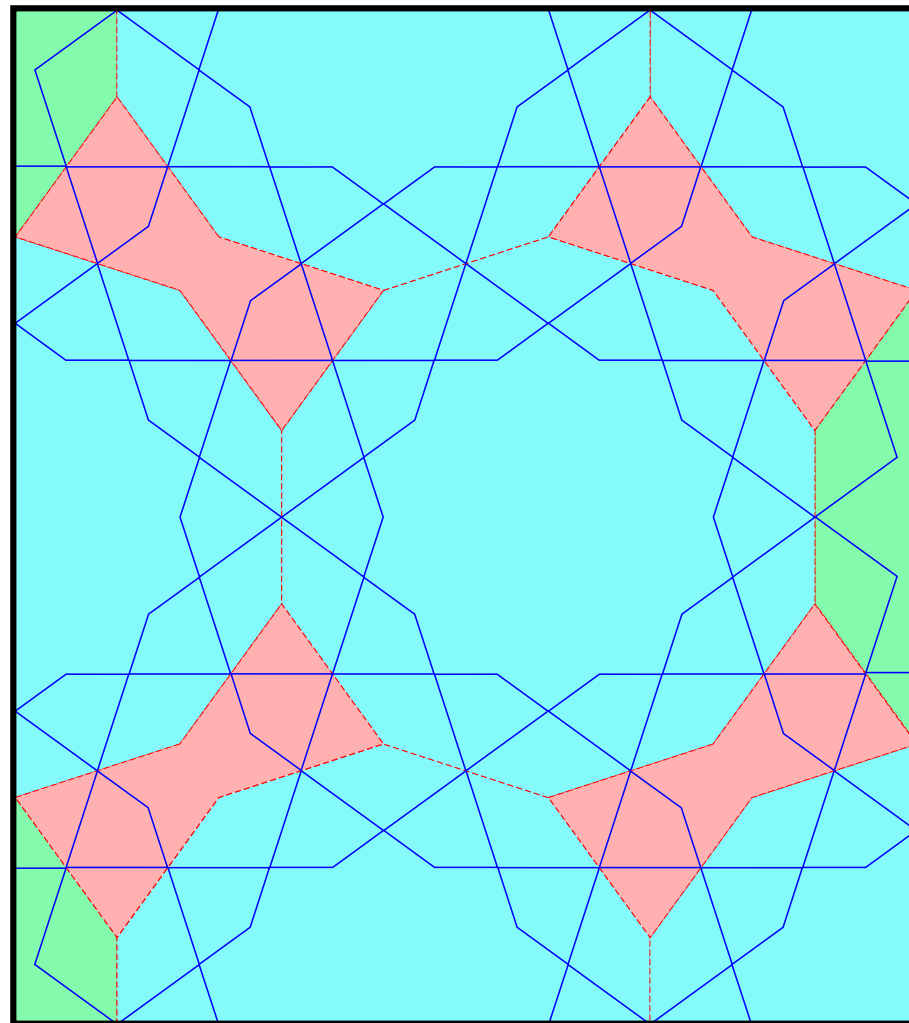
Supplementary Figure S4A Complete panel No. 28 from the Topkapi Scroll, partly shown in Fig. 1G., where the red ink dots mark the boundaries of the girih tiles in Fig. 1F, and which have been filled in with color in the upper reconstruction. The thick red lines correspond to the girih strapwork decoration of girih tiles at a much larger length scale, shown in the lower reconstruction. This is a documented example of girih-tile subdivision, as each large girih tile transforms to the same corresponding pattern of small girih tiles [See main text and note 13.]



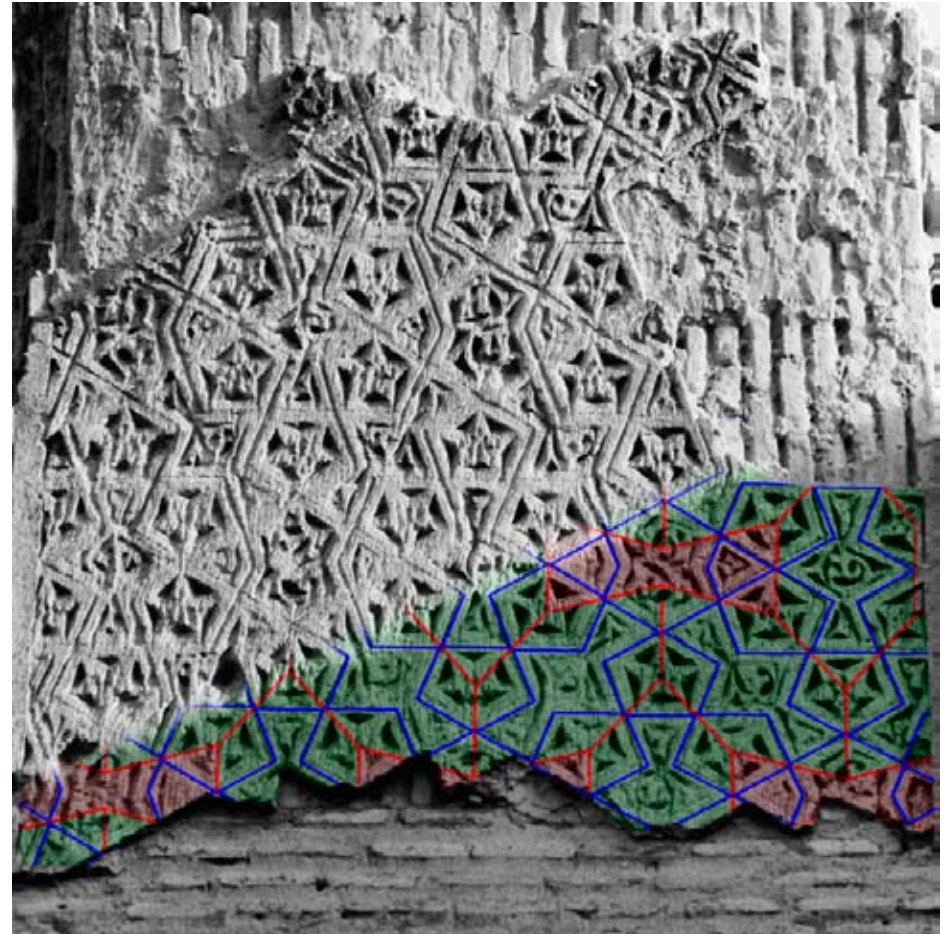
Supplementary Figure S4B Panel No. 50 from the Topkapi Scroll, and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 13.]



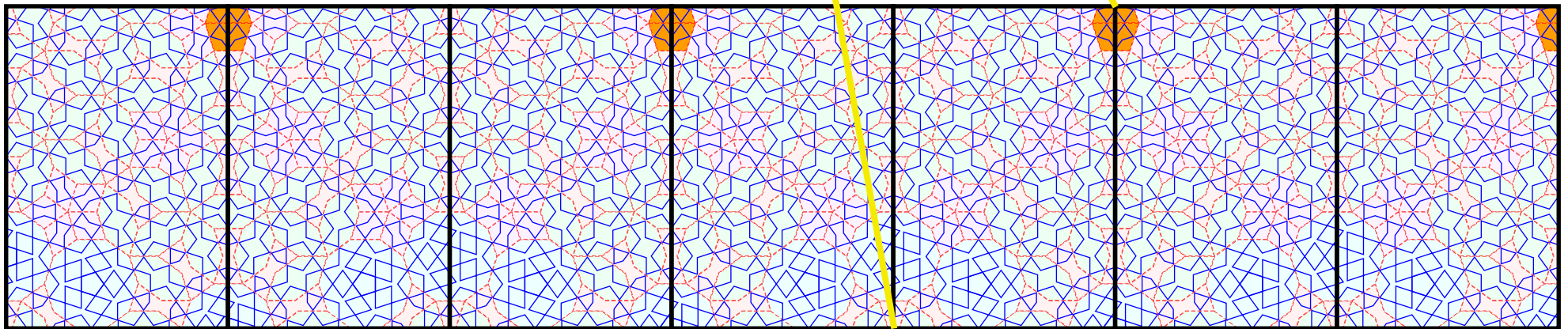
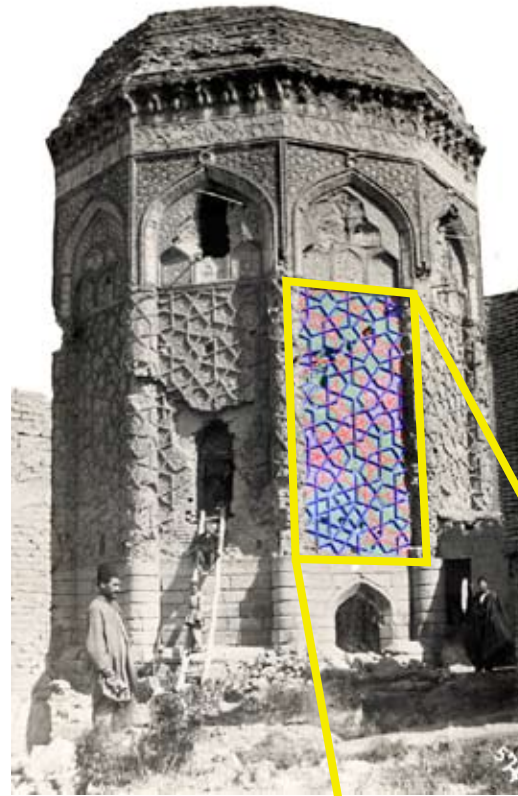
Supplementary Figure S4C Panel No. 52 from the Topkapi Scroll, and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 13.]



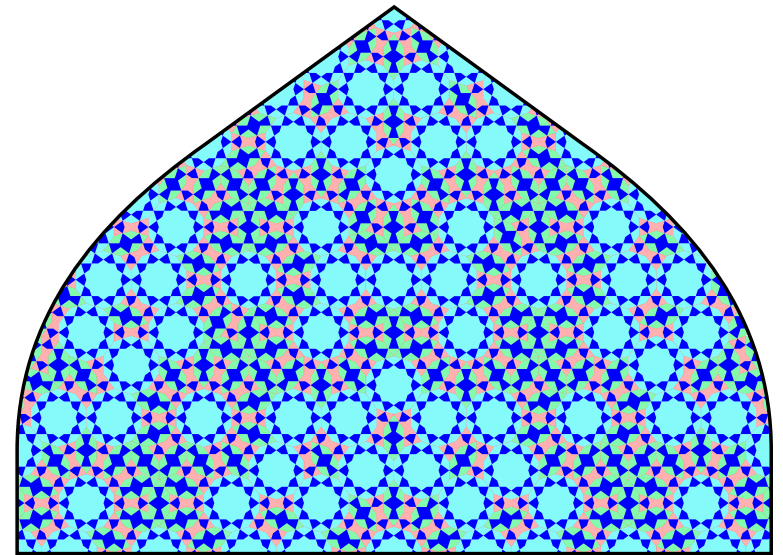
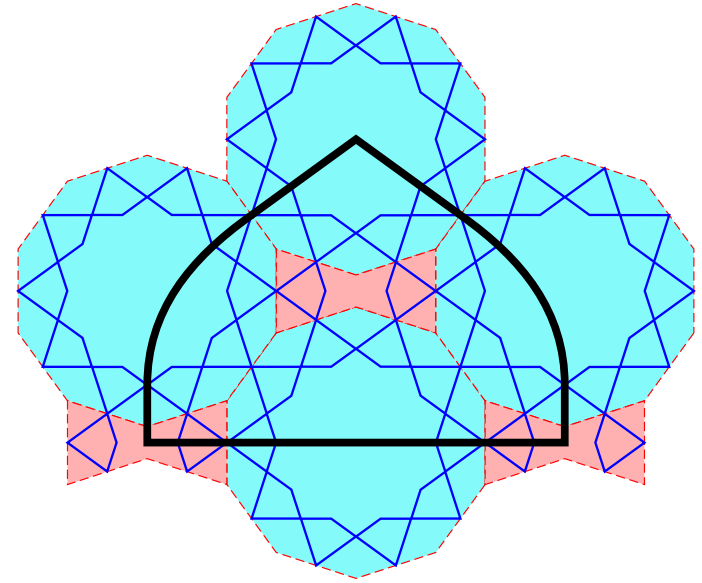
Supplementary Figure S4D Panel from the Tashkent Scroll, and reconstruction of the pattern using the girih tiles of Fig. 1F. [See note 13.]



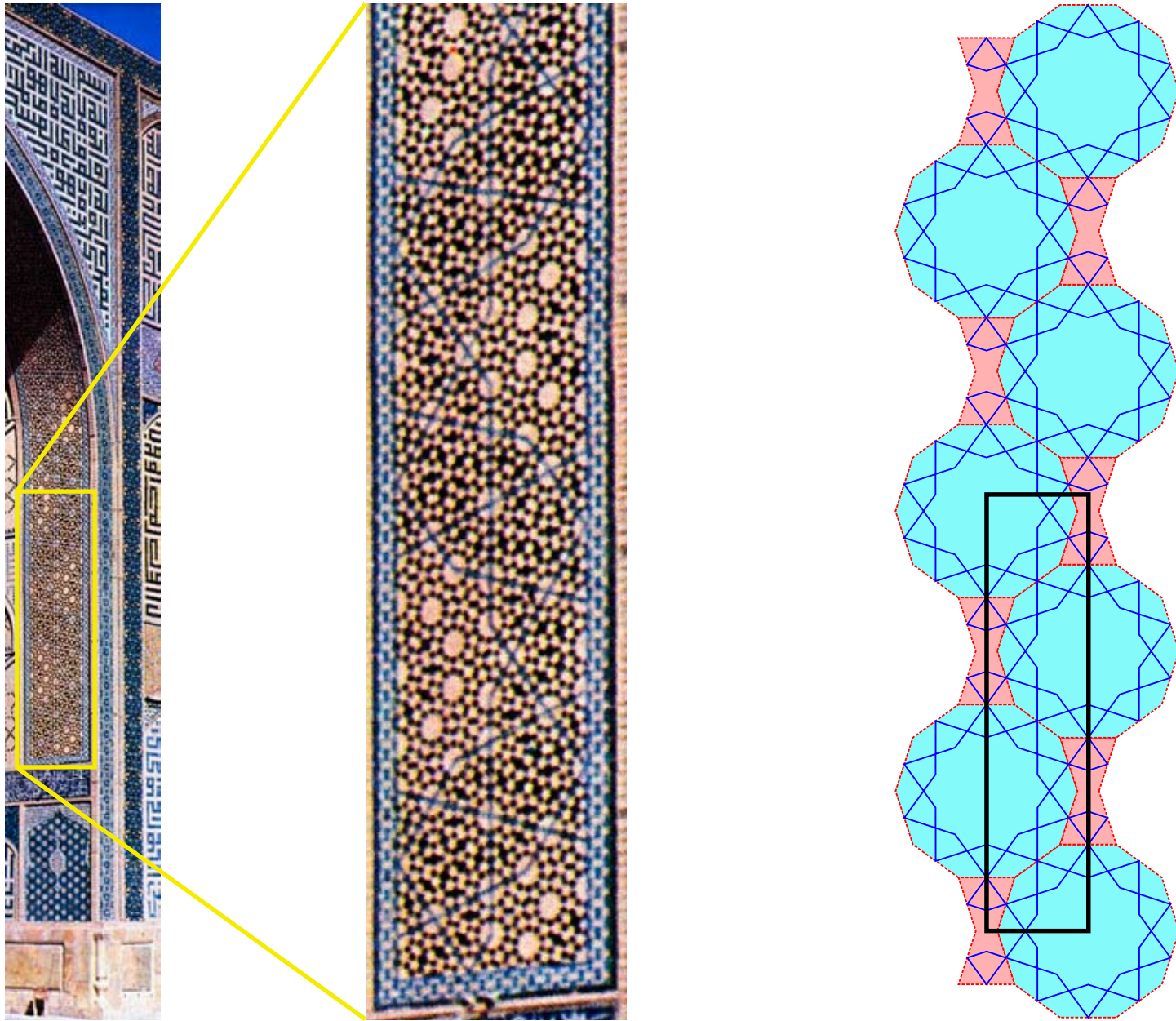
Supplementary Figure S5 (left) Column in the courtyard of the Great Mosque in Malatya, Turkey (c. 1200 AD), and **(right)** the madrasa in Zuzan, Iran (1219 AD), with reconstruction of the pattern using the girih tiles of Fig. 1F overlaid at the bottom of each photograph. [See note 15.]



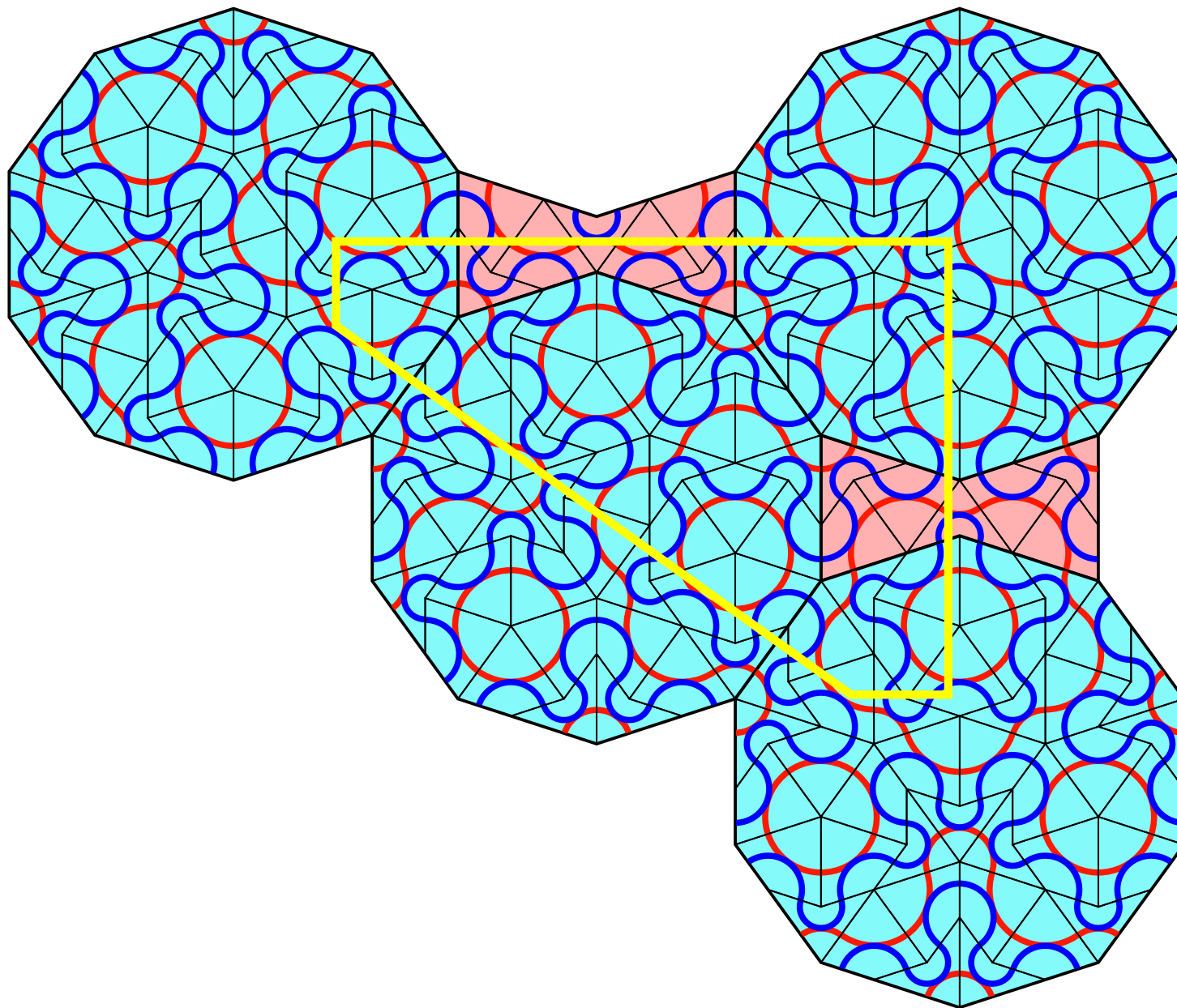
Supplementary Figure S6 (above) Photograph from Fig. 2B. **(below)** Reconstruction of the full pattern covering the seven decorated panels of the octagonal Gunbad-i Kabud in Maragha, Iran (c. 1197 AD) using the girih tiles of Fig. 1F. The area of one panel is denoted by a black rectangle, and covers one of the eight faces of the octagonal tower building (yellow rectangle in upper figure). The girih-polygons perfectly match the brick pattern, except at the corners marked in orange, where two rhombuses have been truncated at the rectangle's edge and the pattern reflected in the adjacent panel. [See note 26].



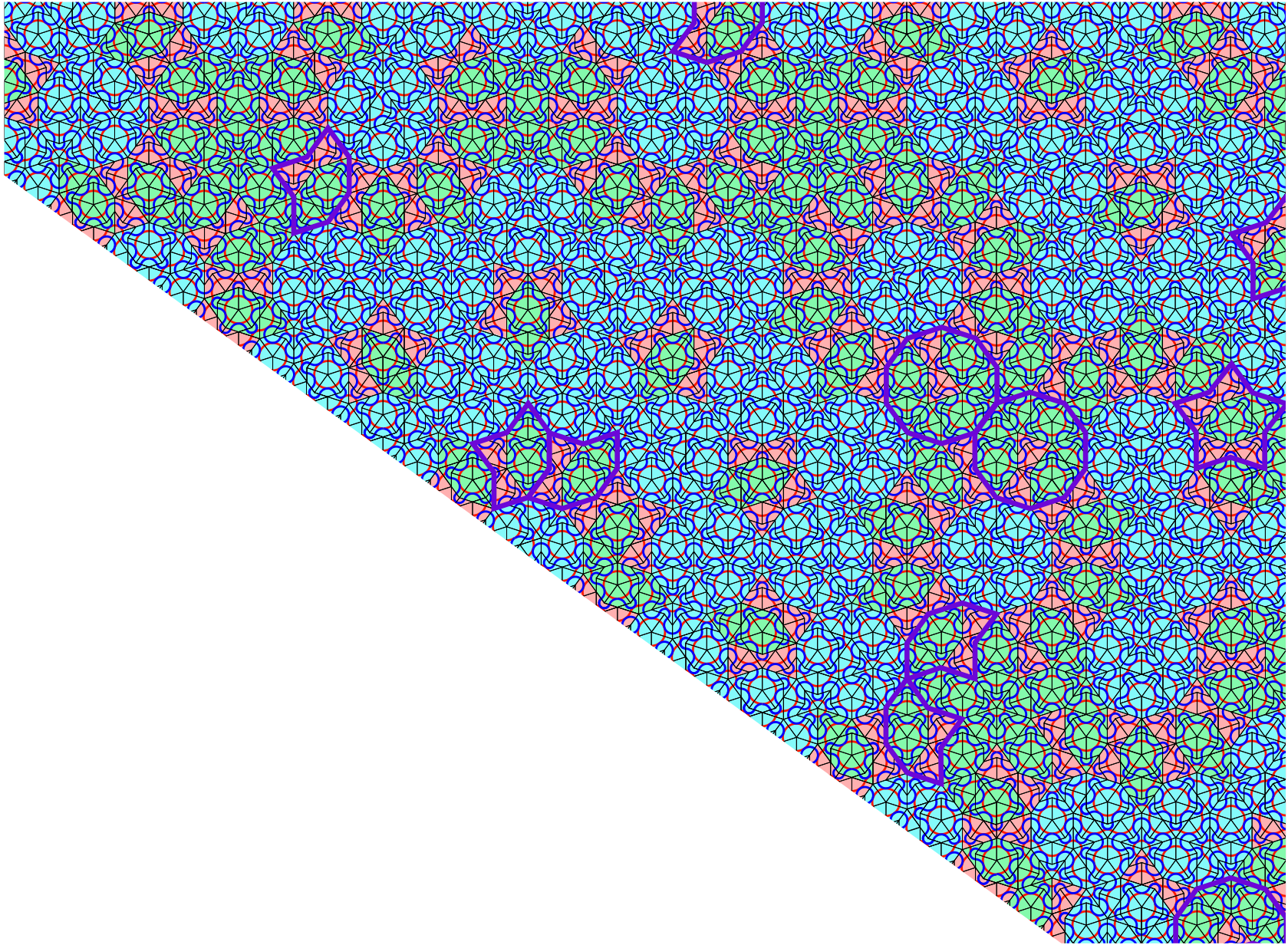
Supplementary Figure S7A Portal from the Darb-i Imam Shrine at Isfahan, Iran (1453 AD). **(left)** Photograph, and reconstructions of the **(upper right)** large and **(lower right)** small girih patterns using the girih tiles of Fig. 1F. The tiling can be subdivided in the same way (Figs. 3D-3E) as the Darb-i Imam pattern in Fig. 3A.



Supplementary Figure S7B Portal in the northwest iwan at the Friday Mosque, Isfahan, Iran (late 15th century AD). **(left)** Photograph, **(center)** enlarged photograph of area enclosed by the yellow rectangle, and **(right)** reconstruction of the large girih-tile pattern using the girih tiles of Fig. 1F. The tiling can be subdivided in the same way (Figs. 3D-3E) as the Darb-i Imam pattern in Fig. 3A.



Supplementary Figure S8A Mapping of the large girih-tile pattern from the spandrel at the Darb-i Imam shrine (Fig. 4C) to a defect-free perfect Penrose tiling fragment, using the mapping given in Figs. 4C-4E.



Supplementary Figure S8B Mapping of the small girih-tile pattern of Fig. 3B to Penrose tiles, using the mapping in Figs. 4C-4E. The purple-bounded regions are local point defects where the Penrose matching rules are violated, and here have been corrected by local tile rearrangements (Fig. 4F) to yield a perfect Penrose tiling here.