Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking

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(Received 25 January 1982)

The treatment of first-order phase transitions for standard grand unified theories is shown to break down for models with radiatively induced spontaneous symmetry breaking. It is argued that proper analysis of these transitions which would take place in the early history of the universe can lead to an explanation of the cosmological homogeneity, flatness, and monopole puzzles.

PACS numbers: 98.80.Bp, 11.15.Ex, 12.10.En

Hot big-bang cosmology depends upon special conditions for the early universe to explain the high degree of homogeneity (the "homogeneity puzzle")[1] and the nearly critical mass density (the "flatness puzzle")[2] found in the universe today. In addition, it has been shown that in typical grand unified theories (GUT's) phase transitions should occur in the early history of the universe which lead to many more magnetic monopoles being produced and surviving to the present epoch than are consistent with experiment (the "monopole puzzle").[3]

In this paper we will argue that first-order phase transitions in a special class of GUT's —models in which the GUT symmetry is broken by radiatively induced corrections to the tree approximation to the effective potential—can lead to a solution to these and other cosmological puzzles. (Models with radiatively induced symmetry breaking, a mechanism discovered by Coleman and Weinberg,[4] will be referred to as CW models.) In particular, we will present results for the standard GUT with a finite-temperature effective (scalar) potential:

$$V_T(\varphi) = (2A - B)\sigma^2\varphi^2 - A\varphi^4 + B\varphi^4 \ln(\varphi^2/\sigma^2) + 18(T^4/\pi^2)\int_0^\infty dx\, x^2 \ln\{1 - \exp[-(x^2 + 25g^2\varphi^2/8T^2)^{1/2}]\}, \tag{1}$$

where the adjoint Higgs field, Φ, has been reexpressed as $\varphi(1,1,1,-\frac{3}{2},-\frac{3}{2})$ (the fundamental Higgs field will be irrelevant for this discussion); g is the gauge coupling constant; σ is chosen to be 4.5×10^{14} GeV; $B = 5625g^4/1024\pi^2$; and A is a free parameter. Equation (1) includes the one-loop quantum and thermal corrections to the effective potential. For a CW model, the coefficient of the quadratic term, $2A - B$, is set equal to zero and the Higgs mass is $m_{\rm CW} = 2.7\times 10^{14}$ GeV. We will also present results for non-CW models in which $2A - B$ is small and therefore the Higgs mass, $m_{\rm H}$, is such that $\Delta_{\rm H} \equiv (m_{\rm H}^2 - m_{\rm CW}^2)/m_{\rm CW}^2$ is small.

As for more general GUT models, the process of the first-order phase transition from the SU(5) *symmetric* phase to the SU(3)⊗SU(2)⊗U(1) *symmetry-breaking* phase for the CW model can be understood by studying the shape of the effective potential as a function of the scalar field for various values of the temperature, as shown in Fig. 1. For temperatures above the critical temperature ($T_{\rm GUT}$) for the transition, the symmetric phase ($\varphi = 0$) is the global stable minimum of the effective potential. At $T = T_{\rm GUT}$, the symmetric phase and the symmetry-breaking phase have equal energy densities. As the temperature drops below $T_{\rm GUT}$, the symmetric phase becomes *metastable*—it has a higher-energy density than *stable* symmetry-breaking phase but a potential barrier prevents it from becoming unstable.

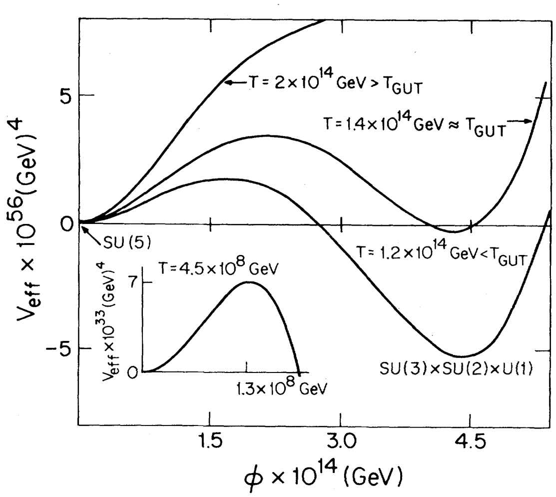

FIG. 1. Effective potential vs φ for various values of T.

The decay of a metastable phase to a stable phase has been compared to a classical nucleation process. At $T < T_{GUT}$ there is a rate per unit time per unit volume, $\Gamma(t)$, for producing finite-sized fluctuations containing stable phase—bubbles—within the metastable system. Once produced, the bubbles grow, coaelesce and convert the system to the stable phase. For cases where the barrier is sufficiently large, Coleman[5] and Linde[6] have found methods for computing $\Gamma(t)$ using a steepest descent (SD) approximation and found it to be of the form $S\exp[-F_f(T)/kT]$. S has the dimensions of (length)$^{-4}$ and k is Boltzmann's constant. $F_f(T)$ is the free energy associated with the bubble computed by SD approximation to be the dominant path across the potential barrier. For this SD bubble fluctuation, the value of $\langle\varphi\rangle$ varies from φ_f (on the stable-phase side of the barrier) in the center of the bubble to $\varphi = 0$ far from the bubble center. As T decreases, $\Gamma(t)$ increases and φ_f/σ decreases, where $\varphi = \sigma$ corresponds to the symmetry-breaking minimum.

The maintenance of a system in a metastable phase during a first-order phase transition as $T \lesssim T_{GUT}$ continues to decrease is known as supercooling. For phase transitions in the early universe, Guth and Tye,[7] taking into account the expansion of the universe, found the expression for the fractional volume $[p(T)]$ of the universe which at temperature T has decayed to the stable phase during supercooling. When $p(T)$, which depends upon $\Gamma(t)$, is of order unity, the decay is said to be *terminated.*

Guth[8] recently suggested that supercooling of first-order phase transitions of typical GUT models can lead to a solution of the cosmological puzzles. His idea was that the energy density of the universe, ρ, during supercooling is dominated (once $T \lesssim T_{GUT}/10$) by the $\rho_0 \approx T_{GUT}^4$, the difference in energy density between the metastable and stable phases. Then, ρ_0 can act as a cosmological constant in Einstein's equation for standard cosmology described by a Robertson-Walker metric:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3M_P^2}\rho \approx \frac{8\pi}{3M_P^2} T_{GUT}^4 \equiv t_{exp}^{-2}, \tag{2}$$

where M_P is the Planck mass. The result is exponential growth of the scale factor, $R(t) \sim R_0 \times \exp(t/t_{exp})$. If the growth is continued long enough, each nearly homogeneous region of the universe experiences an expansion which, Guth showed, could explain the cosmological homogeneity, flatness, and monopole puzzles. The problem with Guth's scenario is that when this high degree of expansion can be arranged in typical GUT's (by adjusting free parameters) the rate of expansion of the universe dominates the rate of production and growth of bubbles; the bubbles never coalesce to complete the transition. Since our own universe exhibits the symmetry breaking of the stable phase, it would have to lie within a single, rare bubble, in which case it is difficult to understand how the high entropy found in our universe could have been generated.[9]

We claim that CW models and near-CW models in which $\Delta_H < 7\times 10^{-6}$ possess special properties that result in completion of the transition to the symmetry-breaking phase along with tremendous expansion. Initially, the analysis of the supercooling for $T < T_{GUT}$ proceeds as in more general GUT models. However, as has been pointed out previously,[9] two important features must be taken into account. Firstly, the GUT fine-structure constant, α ($=\frac{1}{45}$ at $T = T_{GUT}$), increases as a function of temperature[10] until at $T_\alpha \approx 10^6$ GeV, it is of order unity and the one-loop approximation to $V_T(\varphi)$ is no longer valid. Secondly, the prefactor in the expression for $\Gamma(t)$, S, is given by T^4 for CW models since, near $\varphi = 0$, the only parameter with the dimensions of length that affects the barrier and, thus, the decay, is T^{-1}.[9]

When these features are taken into account in the standard SD analysis, they combine to yield the peculiar behavior for $F_f(T)/kT$ and $p(T)$ that is illustrated in Fig. 2. The curves for $F_f(T)/kT$ have been terminated at temperatures, T_{term},

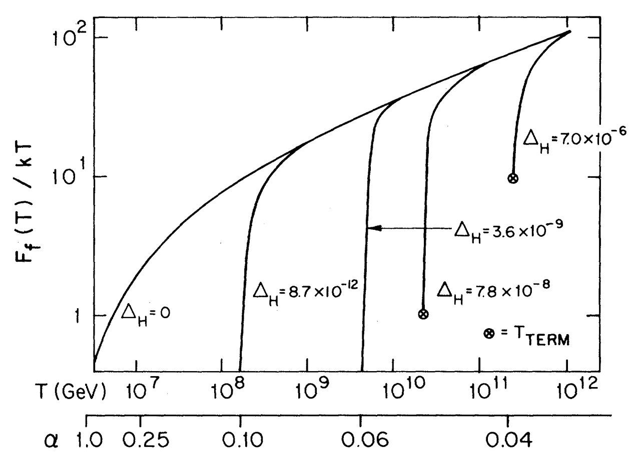

FIG. 2. $F_f(T)/kT$ vs T. Curves terminate for $p(T) \approx 1$. $\Delta_H = (M_H^2 - M_{CW}^2)/M_{CW}^2$.

for which the fractional volume of stable phase, $p(T)$, is of order unity. For $\Delta_H > 7.0 \times 10^{-6}$, $F_f(T_{\rm term})/kT_{\rm term}$ is large, but there is insufficient expansion to solve the cosmological puzzles.

For smaller values of Δ_H, including CW models where $\Delta_H = 0$, $F_f(T_{\rm term})/kT_{\rm term}$ is of order unity or less, values which are too small to trust the SD approximation. The value of the temperature for which the SD approximation fails, $T_{\rm SD}$, is a function of Δ_H and is roughly 10^8 GeV for $\Delta_H = 0$. $T_{\rm SD}$ is larger than T_α in all cases though, so that one can discuss the breakdown of the SD approximation while still considering only the one-loop approximation to the effective potential, Eq. (1).

For the CW and near-CW models, the fact that T reaches $T_{\rm SD}$ before the completion of the transition means that many other types of fluctuations (i.e., paths across the barrier) besides the SD bubble become important. The effect of the barrier becomes negligible. The system (the universe) can be thought of as balancing at a point of unstable (not metastable) equilibrium near $\varphi = 0$. Thermal fluctuations drive different regions of the universe away from the SU(5) symmetric phase but towards different symmetry-breaking minima. Since T is the only dimensional parameter relevant for the fluctuations, the average size of a fluctuation region should be of order T^{-1} and the (roughly constant) value of $\langle\varphi\rangle$ within a fluctuation region is of order T.

Even though $\langle\varphi\rangle \sim T$ corresponds to a point of the effective potential on the stable-phase side of the barrier, slightly to the right of the barrier in Fig. 1, a crucial feature of CW models and near-CW models is that the effective potential is extremely flat from $\varphi = 0$ up to value of $\varphi \sim T_{\rm GUT}$. Thus, even though each fluctuation region has a value of φ that corresponds to a point of classical instability for the effective potential, the motion of φ towards the stable-phase minimum is characterized by a time constant τ that can be very large. Since φ within each fluctuation is of order $T_{\rm SD} \ll T_{\rm GUT}$, the energy density within each fluctuation region is still roughly constant $\sim T_{\rm GUT}^4$. As a result, as concluded independently by Linde,[11] exponential expansion in which φ has a value much less than $T_{\rm GUT}$ continues for a time τ.

We have determined an estimate of τ by considering the evolution according to the classical field equations of a state with $\langle\varphi\rangle = T$ throughout space (presumably a similar method to what was used in Ref. 11) for a range of temperatures for fluctuation production. We found τ for CW models to be large compared to $t_{\rm exp}$. This means that each fluctuation region undergoes many e foldings in spatial expansion before $\langle\varphi\rangle$ changes appreciably. Multiplying the scale factor after time τ by the average size of the initial fluctuation region (the size of an SD bubble was used for $T > T_{\rm SD}$) we obtain the size of the fluctuation region, R_U, at time τ, after which $\langle\varphi\rangle \sim \sigma$ and the exponential expansion ceases. In Table I are shown the results for this computation for a range of temperatures. Column 2 shows R_U(flat) computed with use of the ordinary Klein-Gordon equation. Column 3 shows R_U(exp) derived by using the same equation with an extra drag term $(3\dot{R}\dot{\varphi}/R)$ included to account for the time dependence of the scale factor.[12] Column 4 shows R_U for $\Delta_H = 3.6 \times 10^{-11}$, where the time dependence of the scale factor has been included. For this value of Δ_H, the barrier disappears at $T = 3.7 \times 10^8$ GeV and the maximum value that R_U can achieve is $10^{-0.2}$ cm. Since the observed universe, according to the standard model, had a radius of ~1 cm for $T = 10^{14}$ GeV, the choice of Δ_H must be tuned to a value less than 3.6×10^{-11} in order for the observed universe to fit inside a single fluctuation region. Similarly, we have shown that if our scalar field couples to the curvature through a term $bC\varphi^2$ (C = curvature), $|b|$ must be $\lesssim 10^{-2}$. Please note, we have treated this calculation as if it were in flat space; curvature effects will be discussed in future publications.[12]

The result is that the size of a fluctuation region once $T < T_{\rm SD}$ and the SD approximation breaks down is much greater than the size of our present observed universe (10^{28} cm). If one considers the "observed universe" as lying within such a fluctuation region of the "total universe" the special conditions of hot big-bang cosmology can be satisfied. Because the observed universe would be only a small portion of the total universe result-

TABLE I. Fluctuation radius vs T.

T (GeV)	R_U (flat) (cm)	R_U (exp) (cm)	R_U ($\Delta_H = 3.6 \times 10^{-11}$) (cm)
4.5×10^6	10^{81}	$\gg 10^{470}$	$\cdots$
4.5×10^7	$10^{-4.6}$	10^{470}	$\cdots$
1×10^8	10^{-13}	10^{98}	$\cdots$
3×10^8	10^{-19}	$10^{-4.4}$	$\cdots$
4.5×10^8	10^{-20}	10^{-13}	$10^{-7.2}$
4.5×10^9	10^{-23}	10^{-23}	10^{-23}

ing from the extreme expansion of a small homogeneous region, the homogeneity puzzle is solved. The exponentially large value of scale factor accounts for the flatness puzzle.[8] Because $\langle\varphi\rangle \approx$ const over a fluctuation region, there is no reason to expect to find any monopoles (beyond a small thermally produced number) in the observed universe. Even though there is a discrete symmetry ($\varphi \rightarrow -\varphi$) in the theory, the distance between domain walls (separating regions with $\langle\varphi\rangle$ of opposite sign) produced in the transition should be greater than 10^{28} cm, and hence, unobservable. The potential energy stored in the scalar field is eventually converted to thermal energy,[13] thus producing a sizable entropy density inside each region. Thus, it appears that all the fundamental cosmological puzzles are solved.

A more complete discussion of these results[12] and an analysis of how a fluctuation evolves to thermal equilibrium and produces baryon asymmetry[13] will appear in forthcoming publications.

The authors thank A. Guth and E. Witten for useful discussions and G. Segre for his advice and support during the preparation of the manuscript. This work has been supported in part by the U.S. Department of Energy under Contract No. EY-76-C-02-3071.

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